C&O 781 / QIC 890 Topics in Quantum Information
Recent advances in Quantum Information
University of Waterloo, Fall 2013
Instructor: Ashwin Nayak
Assignment 1, Oct. 4, 2013
Due: by noon, Oct. 18, 2013

Question 1. Consider the "trine states" $|\psi_j\rangle = \cos\frac{2j\pi}{3}|0\rangle + \sin\frac{2j\pi}{3}|1\rangle$, for $j = 0, 1, 2$. Suppose that you are given one of these three states uniformly at random. Use the SDP for State Identification to find the optimal measurement to correctly determine which state is given.

Question 2. Let $\rho_i \in \mathrm{L}(\mathcal{H})$, $1 \leq i \leq n$, be quantum states over a finite dimensional Hilbert space $\mathcal{H}$. We say a measurement $(E_i)_{i=0}^n$ is *unambiguous*, if outcome i is only obtained when state ρ_i is measured. Suppose we are given state ρ_i with probability p_i, $\sum_{i=1}^n p_i = 1$. Formulate the problem of optimally identifying the state with an unambiguous measurement as an SDP. Compute the dual SDP, and present it in the simplest form you can. Does strong duality hold for this SDP?

Question 3. Prove that the strategy we saw in the lectures for Alice in the Spekkens-Rudolph (2002) weak coin-flipping protocol is optimal.

Question 4. Find the optimal cheating probabilities for Alice and Bob in the following weak coin-flipping protocol:

1. Alice prepares the state

$$|\phi\rangle = \frac{1}{\sqrt{2}} \sum_{a\in\{0,1\}} |aa\rangle \otimes |\psi_a\rangle \ , \quad \text{where}$$
$$|\psi_a\rangle = \cos\theta_a\,|00\rangle + \sin\theta_a\,|11\rangle \ ,$$

   and $\theta_a = (-1)^a\theta \in (-\pi/2, \pi/2)$. The state $|\psi\rangle$ is over qubits labeled A, A', S, S'. Alice sends qubit S' to Bob.

2. Bob prepares the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{b\in\{0,1\}} |bb\rangle \ ,$$

   over qubits labeled B, B', and sends qubit B' to Bob.

3. Alice sends qubits A', S to Bob.

4. Alice and Bob measure their qubits according to $(\Pi_{\mathrm{A0}}, \Pi_{\mathrm{A1}}, \Pi_{\mathrm{Aabort}})$, and $\Pi_{\mathrm{B0}}, \Pi_{\mathrm{B1}}, \Pi_{\mathrm{Babort}})$, respec-

tively, where

$$
\begin{aligned}
\Pi_{\mathrm{A0}} &= |00\rangle\langle 00| + |11\rangle\langle 11| \ , \\
\Pi_{\mathrm{A1}} &= |01\rangle\langle 01| + |10\rangle\langle 10| \ , \\
\Pi_{\mathrm{Aabort}} &= 0 \ , \quad \text{and} \\
\Pi_{\mathrm{B0}} &= |00\rangle\langle 00| \otimes |\psi_0\rangle\langle\psi_0| + |11\rangle\langle 11| \otimes |\psi_1\rangle\langle\psi_1| \ , \\
\Pi_{\mathrm{B1}} &= |10\rangle\langle 10| \otimes |\psi_0\rangle\langle\psi_0| + |01\rangle\langle 01| \otimes |\psi_1\rangle\langle\psi_1| \ , \\
\Pi_{\mathrm{Babort}} &= \mathbb{I} - \Pi_{\mathrm{B0}} - \Pi_{\mathrm{B1}} \ ,
\end{aligned}
$$

where Bob's projection operators act on qubits $BA'SS'$.