CO 781 / QIC 823 / CS 867 Quantum Algorithms
University of Waterloo, Winter 2017
Instructor: Ashwin Nayak

Assignment 2, Feb. 13, 2017
Due: by noon, Feb. 27, 2017

Question 1. Prove that any span program for the OR of n bits has complexity at least $\sqrt{n}$.

Question 2. For positive integers n, m such that $m \leq n$, define the threshold function $\mathrm{T}_m^n : \{0,1\}^n \to \{0,1\}$ as $\mathrm{T}_m^n(x) = 1$ if and only if the Hamming weight of x is at least m.

Construct a span program for T_m^n with as small a complexity as you can.

Question 3. Recall the quantum query algorithm and the associated notation corresponding to a given span program. Recall the span program for OR_n with complexity $\sqrt{n}$, and that this is also a span program for NOR_n.

For any 0-input, determine the invariant subspaces of the product of reflections derived from the span program for NOR_n. Verify that the projection of the vector $|0\rangle$ onto eigenspaces with eigen-phases at most $1/100\sqrt{n}$ has norm at most $1/4$ (without relying on the general lemma we proved in this connection).

Repeat the same exercise for OR_n.

Question 4. Consider the Element Distinctness problem, and recall the strategy for solving the problem, as described in Question 1 of Assignment 1. Construct a learning graph corresponding to the strategy with as small a complexity as you can.