

**Comment on “Probabilistic Quantum Memories”**

In a series of similar articles [1–3] Trugenberger claims that quantum states could be used as exponentially large memories for classical information. This claim is wrong. Actually quantum mechanics hardly offers any advantage for this task.

Trugenberger considers an “associative memory” in which exponentially many binary strings are stored. For an additional binary string, the goal is to find whether there are strings close to it (in Hamming distance) in this memory. Also one would like to read out one of these close strings. This might, e.g., be useful to find whether a picture, given a noisy version of it, is in a large database. Trugenberger proposes to use an  $n$ -qubit quantum register as the memory. It is prepared in the uniform amplitude superposition [Eq. (3)] of exponentially many binary strings (here and in the following we refer to [1]). Given the additional string, a sequence of operations is performed which either leads to a measurement result of  $|0\rangle$  (= yes, similar patterns are in the memory) or  $|1\rangle$  (= no, there are no similar patterns). Also, if  $|0\rangle$  is measured, one of the similar strings in the memory is retrieved.

However it is easily seen that a simple classical scheme offers exactly the same performance. Indeed we can replace the  $n$ -qubit memory state with an  $n$ -bit classical memory which stores only a single one of the binary strings. Consider the “processed” memory state [Eq. (16)] just before the measurement. In this state the weights of all “stored” binary strings are still equal, thus no amplification of states close to the additional string has taken place. Because of this, we could simply classically store one of the binary strings, chosen uniformly at random. Then we could compare this random string with the additional one and, depending on how close they are, decide to answer “yes” or “no.” So the  $n$  qubits can be replaced with  $n$  classical bits without changing the result. Of course the performance of such a scheme is very poor, as it really stores only a single bit string. Thus the repeated claims [1–3] of exponential storage capacity or actually of any advantage over classical systems are wrong.

Furthermore the author wrongly assumes that this retrieval step could relatively easily be repeated several times. He states that to this end the “memory state”  $|M\rangle$  could be cloned probabilistically. In [3] he explains that this could be achieved with a “state dependent” cloning machine. Note that a retrieval mechanism which has to know about the data it is supposed to retrieve

contradicts the idea of a memory, whether it is associative or otherwise. In our case the cloning machine would have to know virtually all the information about  $|M\rangle$ , namely, all but possibly  $n$  bits, as a simple argument shows. Indeed to specify the state as opposed to just the set of  $2^n$  linearly independent states to which it belongs (as proposed for the state dependent cloning scheme), one needs at most additional  $n$  bits. Thus the advantage over a simple reparation of  $|M\rangle$  would be marginal (apart from the disadvantage of it being probabilistic, which the author does not discuss). In other words, we would really have to store the whole database classically after all, contrary to the stated goal of the scheme.

Actually it is known that for storing classical information, quantum states in a certain sense cannot offer any advantage. In its simplest version, for perfect channels, the Holevo bound [4] states that a quantum channel is no better than a classical one for transmitting classical information (technically it is a bound on the mutual information). This of course applies just as well to *storing* classical information in quantum bits. A last possible loophole may be better “quantum random access codes.” In these, we could choose which one of a (possibly exponentially) large set of data sets we want to retrieve from a quantum state (whereby the memory would be destroyed). But at least in an asymptotic sense, i.e., large size and high success probability, even this has been ruled out [5]. Thus we strongly doubt that quantum states could be useful as memories for classical information.

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Received 14 March 2003; published 13 November 2003

DOI: 10.1103/PhysRevLett.91.209801

PACS numbers: 03.67.Lx

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