Solving DNN Relaxations of the Quadratic Assignment Problem with ADMM and Facial Reduction

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Motivation

- Semidefinite programming, **SDP**, relaxations extremely strong for many hard discrete optimization problems; particularly true for quadratic assignment problem, **QAP**, one of the hardest NP-complete problems; even finding an ϵ-approximation is NP-complete, Sahni-Gonzales'76. (Nugent instance size $n = 30$ finally solved Anstreicher-Brixius'00 using Condor.)
- difficulties: large dimensional relaxations; inefficiency of the current primal-dual interior point solvers for time and accuracy; high expense in adding cutting plane constraints.
- we propose: alternating direction method of multipliers, **ADMM**, to solve the **SDP** relaxation along with nonnegativity constraints (cuts), i.e., we solve the **DNN** relaxation.
- Several instances are solved to optimality by the relaxation.
- We exploit facial reduction, **FR**; it fits well with **ADMM**.

What is the QAP?

University planning: assign n buildings to n sites

The quadratic assignment problem, **QAP**, in the trace formulation

$$\text{(QAP)} \qquad p^* := \min_{X \in \Pi_n} \langle AXB - 2C, X \rangle,$$

$A, B \in \mathbb{S}^n$ real symmetric $n \times n$ matrices, C real $n \times n$,
$\langle \cdot, \cdot \rangle$ denotes trace inner product, $\langle Y, X \rangle = \operatorname{trace} YX^\top$,
and Π_n set of $n \times n$ permutation matrices (permutations ϕ)

assign n facilities to n locations; minimize total cost

flow is A_{ij} between facilities i, j and it multiplies
distance $B_{\phi(i)\phi(j)}$ to get the total cost of assigning facilities i, j
to locations $\phi(i), \phi(j)$, respectively;
then add location costs in $-\frac{1}{2}\left(C_{i\phi(i)} + C_{j\phi(j)}\right)$

Applications Include: (e.g., Nyberg et al '2012)

Koopmans-Beckmann 1957

- facility location planning: Universities, hospital layout, airport gate assignment, wiring problems/circuit boards/VLSI, typewriter keyboards (though max?)
- Bandwith minimization of a graph
- Image processing
- Scheduling
- Supply Chains
- Economics
- Molecular conformations in chemistry
- Manufacturing lines
- Includes as special case: Traveling salesman problem and Maximum cut problem

Quadratic-Quadratic Model for $X \in \Pi$

$Xe = e, X^T e = e, X \geq 0$, doubly stochastic; $\quad$ (e – ones vector)
turn linear constraints into quadratic

Start with Quadratic-Quadratic Model for $X \in \Pi$, a QQP

$$
\begin{array}{lll}
\min_X & \langle AXB - 2C, X \rangle & \\
\text{s.t.} & \|Xe - e\|^2 + \|X^T e - e\|^2 = 0 & \text{(r-c sums)} \\
& XX^T = X^T X = I_n & \text{(orthogonality)} \\
& X_{ij} X_{ik} = 0, \; X_{ji} X_{ki} = 0, \forall i, \forall j \neq k, & \text{(gangster)} \\
& X_{ij}^2 - X_{ij} = 0, \forall i, j, & (0-1) \\
& X \geq 0 & \text{(nonnegativity)}
\end{array}
$$

Dual of Dual is SDP Relaxation

The Lagrangian dual is an SDP.
The (Lagrangian) dual of this SDP is equivalent to the SDP relaxation of the QQP. BUT, strict feasibility (Slater) fails!

Relaxations for QAP, e.g., Finke-Burkhard-Rendl '87; Rendl-W '87 (and others)

Eigenvalue Bound (apply Hoffman-Wielandt inequality)

$$\begin{array}{rl} \min_X & \langle AXB, X\rangle \\ \text{s.t.} & XX^T = I_n \\ & (X^TX = I_n \text{ NOT redundant in Lagr. relax.}) \end{array}$$

Significantly Stronger Projected Eigenvalue Bound Hadley-Rendl-W. '89

(Used by Brixius-Anstreicher '01 to 'finally' solve Nugent $n = 30$, with help from Condor.)

$$\begin{array}{rl} \min_X & \langle AXB, X\rangle \\ \text{s.t.} & XX^T = I_n, \quad Xe = e, X^Te = e \end{array}$$

More Significantly Stronger but Expensive SDP Bound

FR - SDP bound Zhang-Karisch-Rendl-W. '98

New Derivation of **FR**, **SDP** Relax. in ZKRW, '98

Start new derivation with **QQP** (with fewer constraints)

$$
\begin{array}{lll}
\min_X & \langle AXB - 2C, X\rangle & \\
\text{s.t.} & X_{ij}X_{ik} = 0,\ X_{ji}X_{ki} = 0,\ \forall i,\ \forall j \neq k, & \text{(gangster)} \\
& X_{ij}^2 - X_{ij} = 0,\ \forall i,j, & (0-1) \\
& \sum_{i=1}^n X_{ij}^2 - 1 = 0,\ \forall j,\ \sum_{j=1}^n X_{ij}^2 - 1 = 0,\ \forall i. & \text{(r-c sums)}
\end{array}
$$

Gangster constraints

The first set of constraints, the elementwise orthogonality of the row and columns of X, are the gangster constraints. They are particularly strong constraints and enable many of the other constraints (such as orthogonality $XX^T = I, X^TX = I$, row and columns sums are 1) to be redundant. In fact, after the facial reduction, **FR**, many of these constraints also become redundant.

The Lagrangian Dual (quadratic function!)

Lagrangian

$$\begin{aligned}\mathcal{L}_0(X,U,V,W,u,v) \;=\; & \langle AXB-2C,X\rangle + \\ & \sum_{i=1}^n \sum_{j\neq k} U_{jk}^{(i)} X_{ij}X_{ik} + \\ & \sum_{i=1}^n \sum_{j\neq k} V_{jk}^{(i)} X_{ji}X_{ki} + \\ & \sum_{i,j} W_{ij}(X_{ij}^2 - X_{ij}) + \\ & \sum_{j=1}^n u_j \left(\sum_{i=1}^n X_{ij}^2 - 1\right) + \\ & \sum_{i=1}^n v_i \left(\sum_{j=1}^n X_{ij}^2 - 1\right).\end{aligned}$$

Dual problem is maximization of dual functional d_0

$$\max \; d_0(U,V,W,u,v) := \min_X \mathcal{L}_0(X,U,V,W,u,v)$$

hidden/implicit constraints for inner minimization - Hessian is positive semidefinite, i.e. SDP constraints arise.

Simplify the Dual by Homogenization with x_0

add single constraint $x_0^2 = 1$, add dual variable w_0

$$
\begin{aligned}
\mathcal{L}_1(X, x_0, U, V, W, w_0, u, v) \quad = \quad & y^\top [L_Q + \mathcal{B}_1(U) + \mathcal{B}_2(V) + \\
& \mathrm{Arrow}(w, w_0) + \mathcal{K}_1(u) + \\
& \mathcal{K}_2(v)]\, y - e^\top (u + v) - w_0,
\end{aligned}
$$

where

$$
\mathcal{K}_1(u) = \mathrm{blkdiag}(0, u \otimes I), \quad \mathcal{K}_2(v) = \mathrm{blkdiag}(0, I \otimes v)
$$

$$
\mathrm{Arrow}(w, w_0) = \begin{bmatrix} w_0 & -\frac{1}{2} w^\top \\ -\frac{1}{2} w & \mathrm{Diag}(w) \end{bmatrix}
$$

$$
\mathcal{B}_1(U) = \mathrm{blkdiag}(0, \tilde{U}), \quad \mathcal{B}_2(V) = \mathrm{blkdiag}(0, \tilde{V}).
$$

And, $\tilde{U}$ and $\tilde{V}$ are $n \times n$ block matrices.

SDP Version of Lagrangian Dual

We let $L_Q := \begin{bmatrix} 0 & -\operatorname{vec}(C)^\top \\ -\operatorname{vec}(C) & B \otimes A \end{bmatrix}$.

$$\begin{array}{ll} \max & -e^\top(u+v) - w_0 \\ \text{s.t.} & L_Q + \mathcal{B}_1(U) + \mathcal{B}_2(V) + \operatorname{Arrow}(w, w_0) + \mathcal{K}_1(u) + \mathcal{K}_2(v) \succeq 0. \end{array}$$

where $X \succeq 0$ denotes $X \in \mathbb{S}^n_+$, is $n \times n$ positive semidefinite

Now take the dual of the dual to get **SDP** relaxation.
(Note the similarity to a linear program, LP.)

$$\begin{array}{ll} \min & \operatorname{trace} L_Q Y \\ \text{s.t.} & \operatorname{adjoints}(Y) = \text{RHS} \\ & Y \succeq 0 \end{array}$$

FR: $Y = \widehat{V} R \widehat{V}^T$ Greatly Simplifies **SDP** Relaxation

Dual of Dual after FR

$$
(SDP_R) \qquad \begin{array}{rl} p_R^* := \min_R & \langle L_Q, \widehat{V} R \widehat{V}^\top \rangle \\ \text{s.t.} & \mathcal{G}_J(\widehat{V} R \widehat{V}^\top) = E_{00} \\ & R \succeq 0, \end{array}
$$

- Gangster operator $\mathcal{G}$ *shoots* holes in the matrix $Y = \widehat{V} R \widehat{V}^\top$.
- J is the index set that guarantees the diagonal blocks are diagonal and the off-diagonal blocks have zero diagonal.
- (Some of these block constraints are redundant as are the previous block constraints.)
- with V (now) providing an orthonormal basis for $e^\perp$ and

$$
\widehat{V} = \begin{bmatrix} 1 & 0 \\ \frac{1}{n} e & V \otimes V \end{bmatrix} \qquad \text{(does facial reduction, **FR**)}
$$

Explicit Primal-Dual Strictly Feasible Points

Lemma (ZKRW , Explicit Primal Strictly Feasible Point)

The matrix $\widehat{R}$ defined by

$$\widehat{R} := \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & \frac{1}{n^2(n-1)}\left(nI_{n-1} - E_{n-1}\right) \otimes \left(nI_{n-1} - E_{n-1}\right) \end{array}\right] \in \mathcal{S}_{++}^{(n-1)^2+1}$$

is (strictly) feasible for (SDP_R). □

Dual

Gangster Operator Self-Adjoint, $\mathcal{G}_J^* = \mathcal{G}_J$ (shoots in primal & dual)

Dual Program

$$\begin{array}{rll} d_Y^* := \max\limits_{Y} & \langle E_{00}, Y\rangle & (= Y_{00}) \\ \text{s.t.} & \widehat{V}^\top \mathcal{G}_J(Y)\widehat{V} \preceq \widehat{V}^\top L_Q \widehat{V} & \end{array}$$

Lemma (ZKRW , Explicit Dual Strictly Feasible Point)

The matrices $\widehat{Y}, \widehat{Z}$, *with* $M > 0$ *sufficiently large,*

$$\widehat{Y} := M \left[\begin{array}{c|c} n & 0 \\ \hline 0 & I_n \otimes (I_n - E_n) \end{array}\right] \in \mathcal{S}_{++}^{(n-1)^2+1},$$

$$\widehat{Z} := \widehat{V}^\top L_Q \widehat{V} - \widehat{V}^\top \mathcal{G}_J(\widehat{Y})\widehat{V} \in \mathcal{S}_{++}^{(n-1)^2+1},$$

are (strictly) feasible and slack variables for the dual, respectively.

New **ADMM** Algorithm for the **SDP** Relaxation

KEEP! both Y (for nonneg. and gangster) and R (for $\succeq 0$)

rewrite SDP_R equivalently as

$$\min_{R,Y} \left\{ \langle L_Q, Y \rangle \text{ s.t. } \mathcal{G}_J(Y) = E_{00},\ Y = \widehat{V} R \widehat{V}^\top,\ R \succeq 0 \right\}$$

Therefore we can work with the augmented Lagrangian

$$\mathcal{L}_A(R, Y, Z) = \langle L_Q, Y \rangle + \langle Z, Y - \widehat{V} R \widehat{V}^\top \rangle + \frac{\beta}{2} \| Y - \widehat{V} R \widehat{V}^\top \|_F^2.$$

(R, Y, Z) are the primal reduced, primal, and dual variables, and this denotes the current iterate

$\mathbb{S}^n_{r+}$ denotes the matrices in $\mathbb{S}^n_+$ with rank at most r.

ADMM with Augmented Lagrangian

Updates for (R_+, Y_+, Z_+):

1. $R_+ = \text{argmin}_{R \in \mathbb{S}^n_{r+}} \mathcal{L}_A(R, Y, Z)$
2. $Y_+ = \text{argmin}_{Y \in \mathcal{P}_i} \mathcal{L}_A(R_+, Y, Z)$
3. $Z_+ = Z + \gamma \cdot \beta(Y_+ - \widehat{V} R_+ \widehat{V}^\top)$

Polyhedral Sets

$\mathcal{P}_1 = \{Y \in \mathbb{S}^{n^2+1} : \mathcal{G}_J(Y) = E_{00}\}$ gangster constraints.
$\mathcal{P}_2 = \mathcal{P}_1 \cap \{0 \leq Y \leq 1\}$ (polytope with nonnegativity)

1. Explicit solution for R

Let $\widehat{V}$ be normalized such that $\widehat{V}^\top \widehat{V} = I$. Then:

$$\begin{aligned} R_+ &= \mathrm{argmin}_{R \succeq 0} \langle Z, Y - \widehat{V} R \widehat{V}^\top \rangle + \frac{\beta}{2} \| Y - \widehat{V} R \widehat{V}^\top \|_F^2 \\ &= \mathrm{argmin}_{R \succeq 0} \left\| Y - \widehat{V} R \widehat{V}^\top + \frac{1}{\beta} Z \right\|_F^2 \quad \text{(complete square)} \\ &= \mathrm{argmin}_{R \succeq 0} \left\| R - \widehat{V}^\top (Y + \frac{1}{\beta} Z) \widehat{V} \right\|_F^2 \quad \text{(use } \widehat{V}^\top \widehat{V} = I) \\ &= \mathcal{P}_{\mathbb{S}_+^{(n-1)^2+1}} \left(\widehat{V}^\top (Y + \frac{1}{\beta} Z) \widehat{V} \right), \end{aligned}$$

where we then apply the Eckart-Young-Mirsky Theorem and project onto the face of the **SDP** cone of desired rank ($\leq$ number of positive eigenvalues of the argument).

2. Explicit solution for Y

$i=1$, first linear constraint, Y-subproblem, closed-form solution

$$\begin{aligned} Y_+ &= \operatorname{argmin}_{\mathcal{G}_J(Y)=E_{00}} \langle L_Q, Y\rangle + \langle Z, Y - \widehat{V}R_+\widehat{V}^\top\rangle + \\ &\qquad \tfrac{\beta}{2}\|Y - \widehat{V}R_+\widehat{V}^\top\|_F^2 \\ &= \operatorname{argmin}_{\mathcal{G}_J(Y)=E_{00}} \left\| Y - \widehat{V}R_+\widehat{V}^\top + \frac{L_Q+Z}{\beta}\right\|_F^2 \\ &= E_{00} + \mathcal{G}_{J^c}\left(\widehat{V}R_+\widehat{V}^\top - \frac{L_Q+Z}{\beta}\right). \end{aligned}$$

major advantage of using **ADMM** : we can easily add $0 \le \widehat{V}R\widehat{V}^\top \le 1$ to solve the DNN!:

$$p^*_{RY} := \min_{R,Y}\{\langle L_Q, Y\rangle : \mathcal{G}_J(Y) = E_{00},\ 0 \le Y \le 1,\ Y = \widehat{V}R\widehat{V}^\top,\ R \succeq 0\}$$

Update Y_+ Becomes:

$$Y_+ = E_{00} + \min\left(1, \max\left(0, \mathcal{G}_{J^c}(\widehat{V}R_+\widehat{V}^\top - \tfrac{L_Q+Z}{\beta})\right)\right)$$

Lower bound from Inaccurate Solutions

$(R^{out}, Y^{out}, Z^{out})$ output

Find a lower bound from a feasible solution.
(For inaccurate optimality.)

Lemma

Let $\mathcal{R} := \{R \succeq 0\}$, $\mathcal{Y} := \{Y : \mathcal{G}_J(Y) = E_{00}, 0 \leq Y \leq 1\}$
$\mathcal{Z} := \{Z : \widehat{V}^\top Z \widehat{V} \preceq 0\}$ *and*
$g(Z) := \min_{Y \in \mathcal{Y}} \{\langle L_Q + Z, Y \rangle\}$, *be the* ***ADMM*** *dual function.*
Then the dual of ***ADMM*** *satisfies weak duality and is:*

$$\begin{aligned} d_Z^* &:= \max_{Z \in \mathcal{Z}} g(Z) \\ &\leq p_{RY}^* \quad \textit{(optimal value of } \textbf{\textit{ADMM}}\textit{)} \end{aligned}$$

Proof

Proof.

The dual problem can be derived as

$$\begin{aligned} d_Z^* &:= \max_{Z} \min_{R \in \mathcal{R}, Y \in \mathcal{Y}} \langle L_Q, Y \rangle + \langle Z, Y - \widehat{V} R \widehat{V}^\top \rangle \\ &= \max_{Z} \min_{Y \in \mathcal{Y}} \langle L_Q, Y \rangle + \langle Z, Y \rangle + \min_{R \in \mathcal{R}} \langle Z, -\widehat{V} R \widehat{V}^\top \rangle \\ &= \max_{Z} \min_{Y \in \mathcal{Y}} \langle L_Q, Y \rangle + \langle Z, Y \rangle + \min_{R \in \mathcal{R}} \langle \widehat{V}^\top Z \widehat{V}, -R \rangle \\ &= \max_{Z \in \mathcal{Z}} \min_{Y \in \mathcal{Y}} \langle L_Q + Z, Y \rangle, \\ &= \max_{Z \in \mathcal{Z}} g(Z) \end{aligned}$$

Weak duality follows by exchanging the max and min. □

Implementation: Lower bound from Inaccurate Solutions

- $Z \in \mathcal{Z} \implies g(Z)$ is a lower bound; therefore use projection $g(\mathcal{P}_{\mathcal{Z}}(Z^{out}))$ as lower bound,
- to get $\mathcal{P}_{\mathcal{Z}}(\tilde{Z})$: Let $\bar{V} = (\widehat{V}, \widehat{V}_{\perp})$ be an orthogonal matrix; let $\bar{V}^{\top} Z \bar{V} = W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$; Then:

$$\widehat{V}^{\top} Z \widehat{V} \preceq 0 \Leftrightarrow \widehat{V}^{\top} Z \widehat{V} = \widehat{V}^{\top} \bar{V} W \bar{V}^{\top} \widehat{V} = W_{11} \preceq 0.$$

Hence,

$$\begin{aligned} \mathcal{P}_{\mathcal{Z}}(\tilde{Z}) &= \operatorname{argmin}_{Z \in \mathcal{Z}} \|Z - \tilde{Z}\|_F^2 \\ &= \operatorname{argmin}_{W_{11} \preceq 0} \|\bar{V} W \bar{V}^{\top} - \tilde{Z}\|_F^2 \\ &= \operatorname{argmin}_{W_{11} \preceq 0} \|W - \bar{V}^{\top} \tilde{Z} \bar{V}\|_F^2 \\ &= \begin{bmatrix} \mathcal{P}_{\mathcal{S}_-}(\tilde{W}_{11}) & \tilde{W}_{12} \\ \tilde{W}_{21} & \tilde{W}_{22} \end{bmatrix}, \end{aligned}$$

Upper Bound from Feasible Solution

$(R^{out}, Y^{out}, Z^{out})$ output of ADMM

- obtain best rank-one approximation of Y from largest eigenvalue and corresponding eigenvector: $\lambda vv^\top$.
- Reshape to get square matrix X^{out} as an approximate optimal permutation matrix.
- Since X permutation matrix implies $\operatorname{trace} X^T X = n$, a constant, we get

$$\|X^{out} - X\|_F^2 = -2 \operatorname{trace} X^T X^{out} + \text{ constant.}$$

  Take advantage of the Birkoff, von Neumann Theorem: permutation matrices are extreme points of the doubly stochastic matrices.
- Solve the linear program

$$\max_X \left\{ \langle X^{out}, X \rangle : Xe = e,\ X^\top e = e,\ X \geq 0 \right\}$$

Low Rank Solutions

CHEAT on Rank

Project R onto a rank-one matrix,

$$R_+ = \mathcal{P}_{\mathcal{S}_+^{(n-1)^2+1} \cap \mathcal{R}_1} \left(\widehat{V}^\top (Y + \frac{Z}{\beta}) \widehat{V} \right),$$

where $\mathcal{R}_1 = \{R : \mathrm{rank}(R) = 1\}$ denotes the set of rank-one matrices. For a symmetric matrix W with largest eigenvalue $\lambda > 0$ and corresponding eigenvector w, we have

$$\mathcal{P}_{\mathcal{S}_+^{(n-1)^2+1} \cap \mathcal{R}_1} = \lambda w w^\top.$$

Often provides better feasible solutions/upper bounds.
PROVED OPTIMALITY in 7 instances.

Different choices for $V, \widehat{V}$

matrix $\widehat{V}$ is essential; sparse $\widehat{V}$ helps in projection using a sparse eigenvalue code. From several, the most successful (from Pong, Sun, Wang, W. '14):

$$V = \left[\left[\begin{bmatrix} \left[I_{\lfloor \frac{n}{2} \rfloor} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right] \\ 0_{(n-2\lfloor \frac{n}{2} \rfloor), \lfloor \frac{n}{2} \rfloor} \end{bmatrix} \right] \left[\begin{bmatrix} \left[I_{\lfloor \frac{n}{4} \rfloor} \otimes \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right] \\ 0_{(n-4\lfloor \frac{n}{4} \rfloor), \lfloor \frac{n}{4} \rfloor} \end{bmatrix} \right] \left[\ldots \right] \left[\widehat{V} \right] \right]_{n \times n-1}$$

i.e., the block matrix consisting of t blocks formed from Kronecker products along with one block $\widehat{V}$ to complete the appropriate size so that $V^\top V = I_{n-1}$, $V^\top e = 0$.

Numerical Tests

We used MATLAB version 8.6.0.267246 (R2015b) on a PC Dell Optiplex 9020 64-bit, with 16 Gig, running Windows 7. For the parameters, we heuristically set dual ascent stepsize $\gamma = 1.618$ and quadratic penalty $\beta = \frac{n}{3}$ in **ADMM**. We tested two different stopping tolerances $1e{-}12$ and $1e{-}5$ on:

$$\max\left(\frac{\|Y^k - \widehat{V} R^k \widehat{V}\|_F}{\|Y^k\|_F}, \beta\|Y^{k+1} - Y^k\|_F\right)$$

We tested all **QAP** symmetric instances from QAPLIB with size up to $n = 100$ and compared with Rendl-Sotirov '06 and a standard p-d i-p approach.
(Tests were redone in 2017. Code and results available online.)

QAPLIB: I, bounds - short page 1

	1 opt value	2 Bundle LowBnd	3 HKM-FR LowBnd	4 Tol5 ADMM LowBnd	5 Tol5 feas UpBnd	6 Tol12 ADMM LowBnd	7 Tol12 feas UpBnd	8 Tol5 ADMM %gap	9 ADMM Tol5 vs Bundle %Impr LowBnd
Esc16a	68	59	50	64	78	64	78	20.59	7.35
Esc16b	292	288	276	290	294	290	294	1.37	0.68
Esc16c	160	142	132	154	170	154	170	10.00	7.50
Esc16d	16	8	-12	13	20	13	20	43.75	31.25
Esc16e	28	23	13	27	34	27	34	25.00	14.29
Esc16g	26	20	11	25	34	25	34	34.62	19.23
Esc16h	996	970	909	977	1012	977	1012	3.51	0.70
Esc16i	14	9	-21	12	14	12	14	14.29	21.43
Esc16j	8	7	-4	8	8	8	8	0.00	12.50
Had12	1652	1643	1641	1652	1652	1652	1652	0.00	0.54
Had14	2724	2715	2709	2724	2724	2724	2724	0.00	0.33
Had16	3720	3699	3678	3720	3720	3720	3720	0.00	0.56
Had18	5358	5317	5287	5358	5358	5358	5358	0.00	0.77
Had20	6922	6885	6848	6922	6930	6922	6930	0.12	0.53
Kra30a	88900	77647	-1111	86838	104050	86838	105900	19.36	10.34
Kra30b	91420	81156	-1111	87858	114950	87858	114950	29.63	7.33
Kra32	88700	79659	-1111	85775	111450	85775	111450	28.95	6.90
Nug12	578	557	530	568	654	568	654	14.88	1.90
Nug14	1014	992	960	1011	1022	1011	1022	1.08	1.87
Nug15	1150	1122	1071	1141	1196	1141	1196	4.78	1.65
Nug16a	1610	1570	1528	1600	1610	1600	1610	0.62	1.86
Nug16b	1240	1188	1139	1219	1438	1219	1438	17.66	2.50
Nug17	1732	1669	1622	1708	1756	1708	1756	2.77	2.25
Nug18	1930	1852	1802	1894	2160	1894	2160	13.78	2.18
Nug20	2570	2451	2386	2507	2732	2507	2732	8.75	2.18
Nug21	2438	2323	2386	2382	2672	2382	2672	11.89	2.42
Nug22	3596	3440	3396	3529	3856	3529	3856	9.09	2.47
Nug24	3488	3310	-1111	3402	3658	3402	3658	7.34	2.64
Nug25	3744	3535	-1111	3626	4052	3626	4052	11.38	2.43
Nug27	5234	4965	-1111	5130	5602	5130	5602	9.02	3.15
Nug28	5166	4901	-1111	5026	5534	5026	5534	9.83	2.42
Nug30	6124	5803	-1111	5950	6578	5950	6578	10.25	2.40

QAPLIB: I, bounds - short page 2

	1 opt value	2 Bundle LowBnd	3 HKM-FR LowBnd	4 Tol5 ADMM LowBnd	5 Tol5 feas UpBnd	6 Tol12 ADMM LowBnd	7 Tol12 feas UpBnd	8 Tol5 ADMM %gap	9 ADMM Tol5 vs Bundle %Impr LowBnd
Nug20	2570	2451	2386	2507	2732	2507	2732	8.75	2.18
Nug21	2438	2323	2386	2382	2672	2382	2672	11.89	2.42
Nug22	3596	3440	3396	3529	3856	3529	3856	9.09	2.47
Nug24	3488	3310	-1111	3402	3658	3402	3658	7.34	2.64
Nug25	3744	3535	-1111	3626	4052	3626	4052	11.38	2.43
Nug27	5234	4965	-1111	5130	5602	5130	5602	9.02	3.15
Nug28	5166	4901	-1111	5026	5534	5026	5534	9.83	2.42
Nug30	6124	5803	-1111	5950	6578	5950	6578	10.25	2.40
Rou12	235528	223680	221161	235528	235528	235528	235528	0.00	5.03
Rou15	354210	333287	323235	350217	367782	350217	367782	4.96	4.78
Rou20	725522	663833	642856	695181	765390	695181	765390	9.68	4.32
Scr12	31410	29321	23973	31410	44360	31410	44360	41.23	6.65
Scr15	51140	48836	42204	51140	58304	51140	58304	14.01	4.51
Scr20	110030	94998	83302	106803	149038	106803	149038	38.38	10.73
Tai12a	224416	222784	215637	224416	224416	224416	224416	0.00	0.73
Tai15a	388214	364761	349586	377101	412760	377101	412760	9.19	3.18
Tai17a	491812	451317	441294	476525	546366	476525	546366	14.20	5.13
Tai20a	703482	637300	619092	671675	750450	671676	750450	11.20	4.89
Tai25a	1167256	1041337	-1111	1096657	1271696	1096658	1271696	15.00	4.74
*Tai30a	1818146	1652186	-1111	1706871	1942086	1706872	1942086	12.94	3.01
Tho30	149936	136059	-1111	143576	169958	143576	169958	17.60	5.01

QAPLIB: I, bounds - longer page

	1 opt value	2 Bundle LowBnd	3 HKM-FR LowBnd	4 Tol5 ADMM LowBnd	5 Tol5 feas UpBnd	6 Tol12 ADMM LowBnd	7 Tol12 feas UpBnd	8 Tol5 ADMM %gap	9 ADMM Tol5 vs Bundle %Impr LowBnd
Esc16a	68	59	50	64	78	64	78	20.59	7.35
Esc16b	292	288	276	290	294	290	294	1.37	0.68
Esc16c	160	142	132	154	170	154	170	10.00	7.50
Esc16d	16	8	-12	13	20	13	20	43.75	31.25
Esc16e	28	23	13	27	34	27	34	25.00	14.29
Esc16g	26	20	11	25	34	25	34	34.62	19.23
Esc16h	996	970	909	977	1012	977	1012	3.51	0.70
Esc16i	14	9	-21	12	14	12	14	14.29	21.43
Esc16j	8	7	-4	8	8	8	8	0.00	12.50
Had12	1652	1643	1641	1652	1652	1652	1652	0.00	0.54
Had14	2724	2715	2709	2724	2724	2724	2724	0.00	0.33
Had16	3720	3699	3678	3720	3720	3720	3720	0.00	0.56
Had18	5358	5317	5287	5358	5358	5358	5358	0.00	0.77
Had20	6922	6885	6848	6922	6930	6922	6930	0.12	0.53
Kra30a	88900	77647	-1111	86838	104050	86838	105900	19.36	10.34
Kra30b	91420	81156	-1111	87858	114950	87858	114950	29.63	7.33
Kra32	88700	79659	-1111	85775	111450	85775	111450	28.95	6.90
Nug12	578	557	530	568	654	568	654	14.88	1.90
Nug14	1014	992	960	1011	1022	1011	1022	1.08	1.87
Nug15	1150	1122	1071	1141	1196	1141	1196	4.78	1.65
Nug16a	1610	1570	1528	1600	1610	1600	1610	0.62	1.86
Nug16b	1240	1188	1139	1219	1438	1219	1438	17.66	2.50
Nug17	1732	1669	1622	1708	1756	1708	1756	2.77	2.25
Nug18	1930	1852	1802	1894	2160	1894	2160	13.78	2.18
Nug20	2570	2451	2386	2507	2732	2507	2732	8.75	2.18
Nug21	2438	2323	2386	2382	2672	2382	2672	11.89	2.42
Nug22	3596	3440	3396	3529	3856	3529	3856	9.09	2.47
Nug24	3488	3310	-1111	3402	3658	3402	3658	7.34	2.64
Nug25	3744	3535	-1111	3626	4052	3626	4052	11.38	2.43
Nug27	5234	4965	-1111	5130	5602	5130	5602	9.02	3.15
Nug28	5166	4901	-1111	5026	5534	5026	5534	9.83	2.42
Nug30	6124	5803	-1111	5950	6578	5950	6578	10.25	2.40
Rou12	235528	223680	221161	235528	235528	235528	235528	0.00	5.03
Rou15	354210	333287	323235	350217	367782	350217	367782	4.96	4.78
Rou20	725522	663833	642856	695181	765390	695181	765390	9.68	4.32
Scr12	31410	29321	23973	31410	44360	31410	44360	41.23	6.65
Scr15	51140	48836	42204	51140	58304	51140	58304	14.01	4.51
Scr20	110030	94998	83302	106803	149038	106803	149038	38.38	10.73
Tai12a	224416	222784	215637	224416	224416	224416	224416	0.00	0.73
Tai15a	388214	364761	349586	377101	412760	377101	412760	9.19	3.18
Tai17a	491812	451317	441294	476525	546366	476525	546366	14.20	5.13
Tai20a	703482	637300	619092	671675	750450	671676	750450	11.20	4.89
Tai25a	1167256	1041337	-1111	1096657	1271696	1096658	1271696	15.00	4.74
*Tai30a	1818146	1652186	-1111	1706871	1942086	1706872	1942086	12.94	3.01
Tho30	149936	136059	-1111	143576	169958	143576	169958	17.60	5.01

QAPLIB: I, times/iters - short page

	1 Tol5 cpusec HighRk	2 Tol5 cpusec LowRk	3 HKM cpuratio Tol 9	4 ADMM Tol5 iterations HighRk	5 ADMM Tol5 iterations LowRk	6 ADMM Tol12 iterations HighRk	7 ADMM Tol12 residual HighRk	8 ADMM Tol12 iterations LowRk
Esc16a	20.14	2.64	9.37	2053	280	7309	9.87e-13	305
Esc16b	3.10	2.93	8.08	338	311	641	3.94e-13	334
Esc16c	8.44	3.68	4.88	961	403	3751	9.69e-13	592
Esc16d	17.39	2.18	10.22	1889	236	7812	9.87e-13	270
Esc16e	24.04	2.63	8.79	2719	288	11784	9.93e-13	310
Esc16g	33.54	2.61	8.63	3839	285	9096	9.87e-13	304
Esc16h	4.01	2.73	10.60	433	300	886	8.47e-13	354
Esc16i	100.79	2.26	8.76	11653	290	27106	9.96e-13	323
Esc16j	56.90	2.67	7.93	6898	306	29743	9.95e-13	338
Had12	8.39	0.53	5.91	2682	157	2845	8.64e-13	178
Had14	23.07	0.99	10.46	3919	169	4747	2.35e-13	181
Had16	111.92	1.88	12.51	14179	210	14362	6.80e-13	228
Had18	268.58	3.57	13.28	18068	259	40000	2.07e-06	271
Had20	196.70	6.17	14.53	9038	309	40000	5.55e-07	321
Kra30a	988.47	62.61	-1111	8466	632	40000	2.08e-07	654
Kra30b	1481.32	63.31	-1111	12882	623	40000	8.73e-07	645
Kra32	1355.11	92.43	-1111	9020	720	40000	5.28e-07	737
Nug12	22.27	0.53	5.93	5813	146	40000	3.82e-09	163
Nug14	49.76	1.01	8.43	7667	167	40000	2.94e-07	186
Nug15	53.68	1.49	7.79	6547	200	40000	2.11e-07	221
Nug16a	117.57	1.76	12.24	11591	193	40000	1.46e-06	208
Nug16b	62.72	1.98	11.83	6410	207	40000	5.87e-10	234
Nug17	135.80	2.31	13.13	10727	204	40000	9.12e-07	215
Nug18	250.85	3.22	15.23	15862	226	40000	1.79e-06	240
Nug20	238.68	5.82	14.35	9786	276	40000	4.55e-07	289
Nug21	651.15	8.27	14.95	22465	322	40000	3.62e-06	340
Nug22	942.50	9.84	13.90	27839	325	40000	5.69e-06	338
Nug24	572.04	13.47	-1111	12148	335	40000	7.55e-07	346
Nug25	1308.41	18.38	-1111	24051	375	40000	5.05e-06	386
Nug27	1875.89	30.54	-1111	25201	454	40000	4.16e-06	465
Nug28	1658.48	34.50	-1111	18417	447	40000	2.73e-06	461
Nug30	2584.42	48.92	-1111	22613	469	40000	3.06e-06	478
Rou12	23.19	0.44	6.90	6327	127	6360	2.02e-13	142
Rou15	19.00	1.27	9.46	2219	170	19769	6.08e-13	184
Rou20	88.20	5.60	16.08	3684	263	40000	2.08e-07	275
Scr12	3.71	0.48	5.79	1135	142	2878	6.65e-13	160

QAPLIB: I, times/iters - short page two

	1 Tol5 cpusec HighRk	2 Tol5 cpusec LowRk	3 HKM cpuratio Tol 9	4 ADMM Tol5 iterations HighRk	5 ADMM Tol5 iterations LowRk	6 ADMM Tol12 iterations HighRk	7 ADMM Tol12 residual HighRk	8 ADMM Tol12 iterations LowRk
Nug20	238.68	5.82	14.35	9786	276	40000	4.55e-07	289
Nug21	651.15	8.27	14.95	22465	322	40000	3.62e-06	340
Nug22	942.50	9.84	13.90	27839	325	40000	5.69e-06	338
Nug24	572.04	13.47	-1111	12148	335	40000	7.55e-07	346
Nug25	1308.41	18.38	-1111	24051	375	40000	5.05e-06	386
Nug27	1875.89	30.54	-1111	25201	454	40000	4.16e-06	465
Nug28	1658.48	34.50	-1111	18417	447	40000	2.73e-06	461
Nug30	2584.42	48.92	-1111	22613	469	40000	3.06e-06	478
Rou12	23.19	0.44	6.90	6327	127	6360	2.02e-13	142
Rou15	19.00	1.27	9.46	2219	170	19769	6.08e-13	184
Rou20	88.20	5.60	16.08	3684	263	40000	2.08e-07	275
Scr12	3.71	0.48	5.79	1135	142	2878	6.65e-13	160
Scr15	8.06	1.14	10.75	1061	158	2023	8.11e-13	176
Scr20	858.08	5.94	17.96	34679	264	40000	7.68e-06	276
Tai12a	1.56	0.50	6.70	421	127	454	1.38e-13	145
Tai15a	17.01	1.22	10.34	1955	157	29673	5.41e-13	170
Tai17a	39.60	2.31	12.04	2997	216	22276	7.29e-13	234
Tai20a	66.02	5.62	15.85	2755	252	40000	1.72e-08	267
Tai25a	128.14	17.20	-1111	2244	350	12809	6.33e-13	362
Tai30a	433.54	55.82	-1111	3698	527	39288	3.74e-13	539
Tho30	2045.32	51.37	-1111	17854	522	40000	2.23e-06	533

QAPLIB: I, times/iters - long page

	1 Tol5 cpusec HighRk	2 Tol5 cpusec LowRk	3 HKM cpuratio Tol 9	4 ADMM Tol5 iterations HighRk	5 ADMM Tol5 iterations LowRk	6 ADMM Tol12 iterations HighRk	7 ADMM Tol12 residual HighRk	8 ADMM Tol12 iterations LowRk
Esc16a	20.14	2.64	9.37	2053	280	7309	9.87e-13	305
Esc16b	3.10	2.93	8.08	338	311	641	3.94e-13	334
Esc16c	8.44	3.68	4.88	961	403	3751	9.69e-13	592
Esc16d	17.39	2.18	10.22	1889	236	7812	9.87e-13	270
Esc16e	24.04	2.63	8.79	2719	288	11784	9.93e-13	310
Esc16g	33.54	2.61	8.63	3839	285	9096	9.87e-13	304
Esc16h	4.01	2.73	10.60	433	300	886	8.47e-13	354
Esc16i	100.79	2.26	8.76	11653	290	27106	9.96e-13	323
Esc16j	56.90	2.67	7.93	6898	306	29743	9.95e-13	338
Had12	8.39	0.53	5.91	2682	157	2845	8.64e-13	178
Had14	23.07	0.99	10.46	3919	169	4747	2.35e-13	181
Had16	111.92	1.88	12.51	14179	210	14362	6.80e-13	228
Had18	268.58	3.57	13.28	18068	259	40000	2.07e-06	271
Had20	196.70	6.17	14.53	9038	309	40000	5.55e-07	321
Kra30a	988.47	62.61	-1111	8466	632	40000	2.08e-07	654
Kra30b	1481.32	63.31	-1111	12882	623	40000	8.73e-07	645
Kra32	1355.11	92.43	-1111	9020	720	40000	5.28e-07	737
Nug12	22.27	0.53	5.93	5813	146	40000	3.82e-09	163
Nug14	49.76	1.01	8.43	7667	167	40000	2.94e-07	186
Nug15	53.68	1.49	7.79	6547	200	40000	2.11e-07	221
Nug16a	117.57	1.76	12.24	11591	193	40000	1.46e-06	208
Nug16b	62.72	1.98	11.83	6410	207	40000	5.87e-10	234
Nug17	135.80	2.31	13.13	10727	204	40000	9.12e-07	215
Nug18	250.85	3.22	15.23	15862	226	40000	1.79e-06	240
Nug20	238.68	5.82	14.35	9786	276	40000	4.55e-07	289
Nug21	651.15	8.27	14.95	22465	322	40000	3.62e-06	340
Nug22	942.50	9.84	13.90	27839	325	40000	5.69e-06	338
Nug24	572.04	13.47	-1111	12148	335	40000	7.55e-07	346
Nug25	1308.41	18.38	-1111	24051	375	40000	5.05e-06	386
Nug27	1875.89	30.54	-1111	25201	454	40000	4.16e-06	465
Nug28	1658.48	34.50	-1111	18417	447	40000	2.73e-06	461
Nug30	2584.42	48.92	-1111	22613	469	40000	3.06e-06	478
Rou12	23.19	0.44	6.90	6327	127	6360	2.02e-13	142
Rou15	19.00	1.27	9.46	2219	170	19769	6.08e-13	184
Rou20	88.20	5.60	16.08	3684	263	40000	2.08e-07	275
Scr12	3.71	0.48	5.79	1135	142	2878	6.65e-13	160
Scr15	8.06	1.14	10.75	1061	158	2023	8.11e-13	176
Scr20	858.08	5.94	17.96	34679	264	40000	7.68e-06	276
Tai12a	1.56	0.50	6.70	421	127	454	1.38e-13	145
Tai15a	17.01	1.22	10.34	1955	157	29673	5.41e-13	170
Tai17a	39.60	2.31	12.04	2997	216	22276	7.29e-13	234
Tai20a	66.02	5.62	15.85	2755	252	40000	1.72e-08	267
Tai25a	128.14	17.20	-1111	2244	350	12809	6.33e-13	362
Tai30a	433.54	55.82	-1111	3698	527	39288	3.74e-13	539
Tho30	2045.32	51.37	-1111	17854	522	40000	2.23e-06	533

QAPLIB II - short page

	1. opt value	2. ADMM LowBnd	3. feas UpBnd	4. ADMM %gap	5 Tol5 cpusec HighRk	6 Tol5 cpusec LowRk	7 ADMM iterations HighRk	8 ADMM iterations LowRk
Chr12a	9552	0	9552	100.00	6.53e+01	4.08e-01	21061	117
Chr12b	9742	0	9742	100.00	3.32e+01	4.11e-01	10592	119
Chr12c	11156	0	11156	100.00	7.42e+01	3.96e-01	23982	115
Chr15a	9896	0	9896	100.00	2.07e+02	1.28e+00	31937	173
Chr15b	7990	0	7990	100.00	2.69e+01	9.84e-01	3976	133
Chr15c	9504	0	9504	100.00	1.54e+01	1.06e+00	2192	147
Chr18a	11098	1	11098	99.99	4.94e+02	2.86e+00	40000	198
Chr18b	1534	0	2264	147.59	5.72e+01	3.08e+00	3843	243
Chr20a	2192	1	2192	99.95	7.40e+02	4.31e+00	40000	217
Chr20b	2298	0	2298	100.00	1.42e+02	5.31e+00	6355	243
Chr20c	14142	1	14142	99.99	7.28e+02	5.03e+00	40000	232
Chr22a	6156	0	6156	100.00	4.02e+02	9.37e+00	14051	310
Chr22b	6194	0	6194	100.00	3.80e+02	9.45e+00	11418	304
Chr25a	3796	0	3796	100.00	3.06e+02	1.70e+01	6164	355
Els19	17212548	1	17212548	100.00	6.17e+02	4.48e+00	40000	269
Esc16f	0	1	0	-Inf	3.22e+02	3.39e+02	40000	40000
Esc32a	130	0	168	129.23	2.89e+03	9.16e+01	20398	700
Esc32b	168	0	264	157.14	2.52e+03	8.31e+01	17920	658
Esc32c	642	0	686	106.85	4.48e+02	1.01e+02	3177	780
Esc32d	200	0	228	114.00	8.68e+02	1.09e+02	6334	825
Esc32e	2	0	2	100.00	1.81e+03	1.05e+02	13040	836
Esc32f	2	0	2	100.00	1.80e+03	1.07e+02	13040	836
Esc32g	6	0	8	133.33	6.04e+02	1.06e+02	4405	855
Esc32h	438	0	482	110.05	3.02e+03	1.00e+02	21515	795
*Sko42	15812	0	17086	108.06	1.06e+04	3.87e+02	21013	911
*Sko49	23386	0	25076	107.23	3.03e+04	1.18e+03	28771	1316
*Sko56	34458	0	36580	106.16	3.90e+04	2.68e+03	21106	1664
Ste36a	9526	1	13866	145.55	1.02e+04	1.87e+02	40000	851
Ste36b	15852	1	25878	163.24	1.01e+04	1.56e+02	40000	700
Ste36c	8239110	1	11152926	135.37	1.01e+04	1.69e+02	40000	798
*Tai35a	2422002	0	2599924	107.35	7.40e+02	1.33e+02	3225	661
*Tai40a	3139370	0	3392692	108.07	1.94e+03	2.99e+02	4665	852
*Tai50a	4938796	0	5332790	107.98	6.36e+03	1.33e+03	5393	1348
*Tho40	240516	0	269452	112.03	8.52e+03	2.90e+02	21131	828
*Wil50	48816	0	50040	102.51	1.73e+04	1.43e+03	15370	1473
Esc64a	116	1	120	102.59	1.64e+04	1.11e+04	2000	2000
*Sko64	48498	1	50840	104.83	1.56e+04	1.13e+04	2000	2000
*Sko72	66256	1	70672	106.66	3.01e+04	2.07e+04	2000	2000

QAPLIB II - short page two

	1. opt value	2. ADMM LowBnd	3. feas UpBnd	4. ADMM %gap	5 Tol5 cpusec HighRk	6 Tol5 cpusec LowRk	7 ADMM iterations HighRk	8 ADMM iterations LowRk
*Sko42	15812	0	17086	108.06	1.06e+04	3.87e+02	21013	911
*Sko49	23386	0	25076	107.23	3.03e+04	1.18e+03	28771	1316
*Sko56	34458	0	36580	106.16	3.90e+04	2.68e+03	21106	1664
Ste36a	9526	1	13866	145.55	1.02e+04	1.87e+02	40000	851
Ste36b	15852	1	25878	163.24	1.01e+04	1.56e+02	40000	700
Ste36c	8239110	1	11152926	135.37	1.01e+04	1.69e+02	40000	798
*Tai35a	2422002	0	2599924	107.35	7.40e+02	1.33e+02	3225	661
*Tai40a	3139370	0	3392692	108.07	1.94e+03	2.99e+02	4665	852
*Tai50a	4938796	0	5332790	107.98	6.36e+03	1.33e+03	5393	1348
*Tho40	240516	0	269452	112.03	8.52e+03	2.90e+02	21131	828
*Wil50	48816	0	50040	102.51	1.73e+04	1.43e+03	15370	1473
Esc64a	116	1	120	102.59	1.64e+04	1.11e+04	2000	2000
*Sko64	48498	1	50840	104.83	1.56e+04	1.13e+04	2000	2000
*Sko72	66256	1	70672	106.66	3.01e+04	2.07e+04	2000	2000
*Sko81	90998	1	96456	106.00	5.94e+04	3.77e+04	2000	2000
*Sko90	115534	1	121390	105.07	9.32e+04	6.72e+04	2000	2000
*Sko100a	152002	1	160794	105.78	1.38e+05	9.37e+04	2000	2000
*Sko100b	153890	1	162004	105.27	1.38e+05	9.45e+04	2000	2000
*Sko100c	147862	1	156230	105.66	1.38e+05	9.46e+04	2000	2000
*Sko100d	149576	1	157100	105.03	1.39e+05	9.53e+04	2000	2000
*Sko100e	149150	1	155858	104.50	1.38e+05	9.51e+04	2000	2000
*Sko100f	149036	1	0	-0.00	7.68e+04	0.00e+00	1100	0
*Tai60a	7205962	1	7759332	107.68	1.34e+04	1.01e+04	2000	1965
Tai64c	1855928	1	1917484	103.32	1.65e+04	1.14e+04	2000	2000
*Tai80a	13499184	1	14618694	108.29	5.17e+04	3.08e+04	2000	2000
*Tai100	21052466	1	22641778	107.55	1.53e+05	9.33e+04	2000	2000
*Wil100	273038	1	278898	102.15	1.41e+05	9.67e+04	2000	2000

QAPLIB II - long page

	1. opt value	2. ADMM LowBnd	3. feas UpBnd	4. ADMM %gap	5 Tol5 cpusec HighRk	6 Tol5 cpusec LowRk	7 ADMM iterations HighRk	8 ADMM iterations LowRk
Chr12a	9552	0	9552	100.00	6.53e+01	4.08e-01	21061	117
Chr12b	9742	0	9742	100.00	3.32e+01	4.11e-01	10592	119
Chr12c	11156	0	11156	100.00	7.42e+01	3.96e-01	23982	115
Chr15a	9896	0	9896	100.00	2.07e+02	1.28e+00	31937	173
Chr15b	7990	0	7990	100.00	2.69e+01	9.84e-01	3976	133
Chr15c	9504	0	9504	100.00	1.54e+01	1.06e+00	2192	147
Chr18a	11098	1	11098	99.99	4.94e+02	2.86e+00	40000	198
Chr18b	1534	0	2264	147.59	5.72e+01	3.08e+00	3843	243
Chr20a	2192	1	2192	99.95	7.40e+02	4.31e+00	40000	217
Chr20b	2298	0	2298	100.00	1.42e+02	5.31e+00	6355	243
Chr20c	14142	1	14142	99.99	7.28e+02	5.03e+00	40000	232
Chr22a	6156	0	6156	100.00	4.02e+02	9.37e+00	14051	310
Chr22b	6194	0	6194	100.00	3.80e+02	9.45e+00	11418	304
Chr25a	3796	0	3796	100.00	3.06e+02	1.70e+01	6164	355
Els19	17212548	1	17212548	100.00	6.17e+02	4.48e+00	40000	269
Esc16f	0	1	0	-Inf	3.22e+02	3.39e+02	40000	40000
Esc32a	130	0	168	129.23	2.89e+03	9.16e+01	20398	700
Esc32b	168	0	264	157.14	2.52e+03	8.31e+01	17920	658
Esc32c	642	0	686	106.85	4.48e+02	1.01e+02	3177	780
Esc32d	200	0	228	114.00	8.68e+02	1.09e+02	6334	825
Esc32e	2	0	2	100.00	1.81e+03	1.05e+02	13040	836
Esc32f	2	0	2	100.00	1.80e+03	1.07e+02	13040	836
Esc32g	6	0	8	133.33	6.04e+02	1.06e+02	4405	855
Esc32h	438	0	482	110.05	3.02e+03	1.00e+02	21515	795
*Sko42	15812	0	17086	108.06	1.06e+04	3.87e+02	21013	911
*Sko49	23386	0	25076	107.23	3.03e+04	1.18e+03	28771	1316
*Sko56	34458	0	36580	106.16	3.90e+04	2.68e+03	21106	1664
Ste36a	9526	1	13866	145.55	1.02e+04	1.87e+02	40000	851
Ste36b	15852	1	25878	163.24	1.01e+04	1.56e+02	40000	700
Ste36c	8239110	1	11152926	135.37	1.01e+04	1.69e+02	40000	798
*Tai35a	2422002	0	2599924	107.35	7.40e+02	1.33e+02	3225	661
*Tai40a	3139370	0	3392692	108.07	1.94e+03	2.99e+02	4665	852
*Tai50a	4938796	0	5332790	107.98	6.36e+03	1.33e+03	5393	1348
*Tho40	240516	0	269452	112.03	8.52e+03	2.90e+02	21131	828
*Wil50	48816	0	50040	102.51	1.73e+04	1.43e+03	15370	1473
Esc64a	116	1	120	102.59	1.64e+04	1.11e+04	2000	2000
*Sko64	48498	1	50840	104.83	1.56e+04	1.13e+04	2000	2000
*Sko72	66256	1	70672	106.66	3.01e+04	2.07e+04	2000	2000
*Sko81	90998	1	96456	106.00	5.94e+04	3.77e+04	2000	2000
*Sko90	115534	1	121390	105.07	9.32e+04	6.72e+04	2000	2000
*Sko100a	152002	1	160794	105.78	1.38e+05	9.37e+04	2000	2000
*Sko100b	153890	1	162004	105.27	1.38e+05	9.45e+04	2000	2000
*Sko100c	147862	1	156230	105.66	1.38e+05	9.46e+04	2000	2000
*Sko100d	149576	1	157100	105.03	1.39e+05	9.53e+04	2000	2000
*Sko100e	149150	1	155858	104.50	1.38e+05	9.51e+04	2000	2000
*Sko100f	149036	1	0	-0.00	7.68e+04	0.00e+00	1100	0
*Tai60a	7205962	1	7759332	107.68	1.34e+04	1.01e+04	2000	1965
Tai64c	1855928	1	1917484	103.32	1.65e+04	1.14e+04	2000	2000
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*Tai100	21052466	1	22641778	107.55	1.53e+05	9.33e+04	2000	2000
*Wil100	273038	1	278898	102.15	1.41e+05	9.67e+04	2000	2000

Conclusion

- We presented ADMM framework for QAP that exploits facial reduction. (Keeping both R and Y appears to be advantageous.)
- solve large problems to extremely high accuracy while solving the DNN relaxation. This yielded improved bounds. Several problems were solved to optimality.
- The ADMM approach together with facial reduction appears to be very promising.
  In progress (with - you know who you are): success for graph partitioning, second lifting of max-cut, and a special case of the min-cut problem.

Thanks for your attention!

Solving DNN Relaxations of the Quadratic Assignment Problem with ADMM and Facial Reduction

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