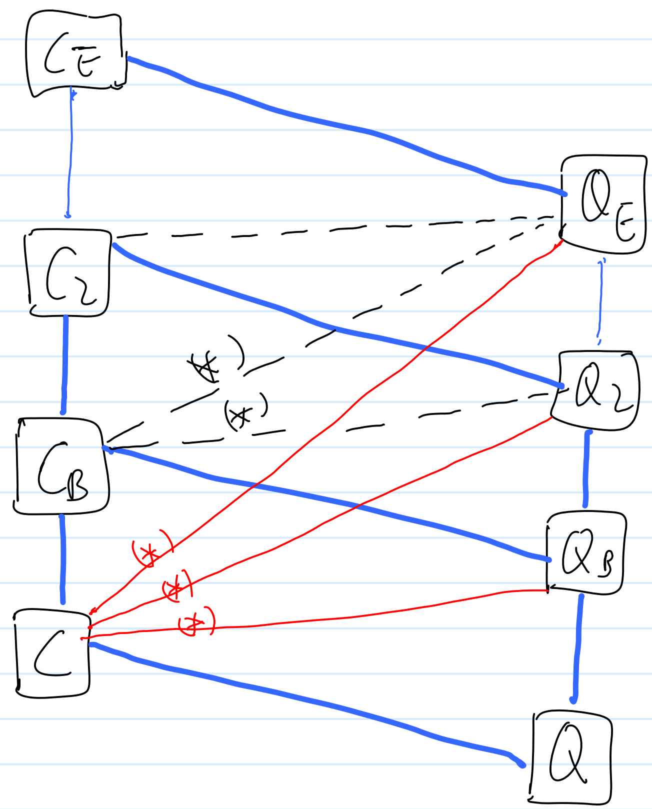
Lec 21, July 20, 2010.

Note Title 20/07/2010

For any 2 capacities
are they related by "$\leq$" $\forall N$
or are they incomparable,
or we don't know?



0406086 Bennett, Devetak, Shor, Smolin
0904.4050 Smith, Smolin.

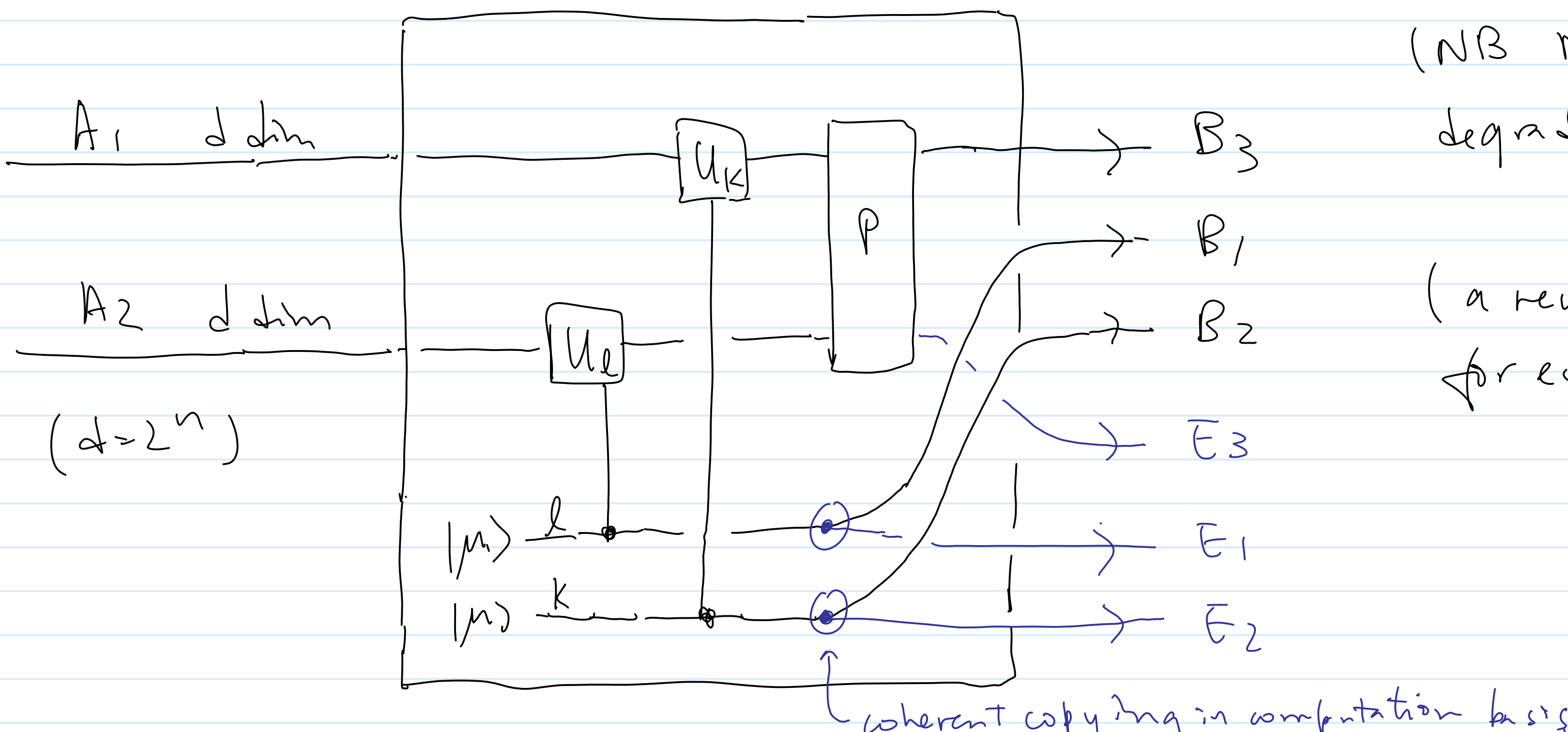
(*) R_d & classical channel.

C small but Q_B Q_2 Q_E all large.

$C_E = 1$ $\therefore Q_E = \frac{1}{2}$

But $C = 1$, $Q \leq Q_B \leq Q_2 \leq Q_E = \frac{1}{2}$

The Rocket channel R_d (iso ext):



(NB Neither degradable (anti deg).

(a new (ℓ, k) is drawn for every use of R_d)

$$|\mu\rangle = \frac{1}{\sqrt{|C_n|}} \sum_{\ell=1}^{|C_n|} |\ell\rangle , \quad P\,|i\rangle_{A_1} |j\rangle_{A_2} = \omega^{ij}\, |i\rangle_{A_1} |j\rangle_{A_2}$$

C_n : Clifford group on n qubits, ω = primitive dth root of unity

Properties of R_d to be shown / discussed :

① $C(R_d) \leq 2$ $\quad$ ¡¡ $\quad \underbrace{Q(R_d) \leq P(R_d) \leq C(R_d)}_{\text{general ineq } \forall \text{ channel}} \leq 2$

② $Q_E(R_d) \geq \log d$, $\quad$ ¡¡ $C_E(R_d) \geq 2\log d$

$P_E(R_d) \geq 2\log d$

③ $Q^{(1)}(R_d \otimes E_{\frac{1}{2}}) \geq \frac{1}{2}\log d$ $\quad$ ¡¡ $Q_{ss}(R_d) \geq \frac{1}{2}\log d$

④ $Q_2(R_d) \geq \log d$

$Q_B(R_d) \geq \frac{1}{2}\log d$

$C_B(R_d) \geq \frac{2}{3}\log d$

(1) The channel is designed to suppress Q & C :

- U_ℓ unknown to sender

  Else sender can input $U_\ell^{-1}|0\rangle$ to A_2

  and $A_1 \to B_3$ will then be a noiseless Q channel, $Q(R_d) \geq \log d$

- U_K unknown to sender

  Else $U_K^{-1}|i\rangle$ for $i = 1, \dots, d$ will emerge as phase $\times\, |i\rangle$

  and $A_1 \to B_3$ will be a noiseless C classical, $C(R_d) \geq \log d$

- See 0904.4050 for a proof why $C \leq 2$.

  Main idea: $\forall$ input, averaged over ℓ, k, $B_3^{\otimes n}$ & $E_3^{\otimes n}$ very entangled

  This can be rigorously proved: $\forall \rho$ $\quad S(R_d^{\otimes n}(\rho)) \geq n(\log d - 2)$

  So an upper bound $C(R_d) \leq 2$ (even if $\mathcal{X}^{(1)} \neq C$).

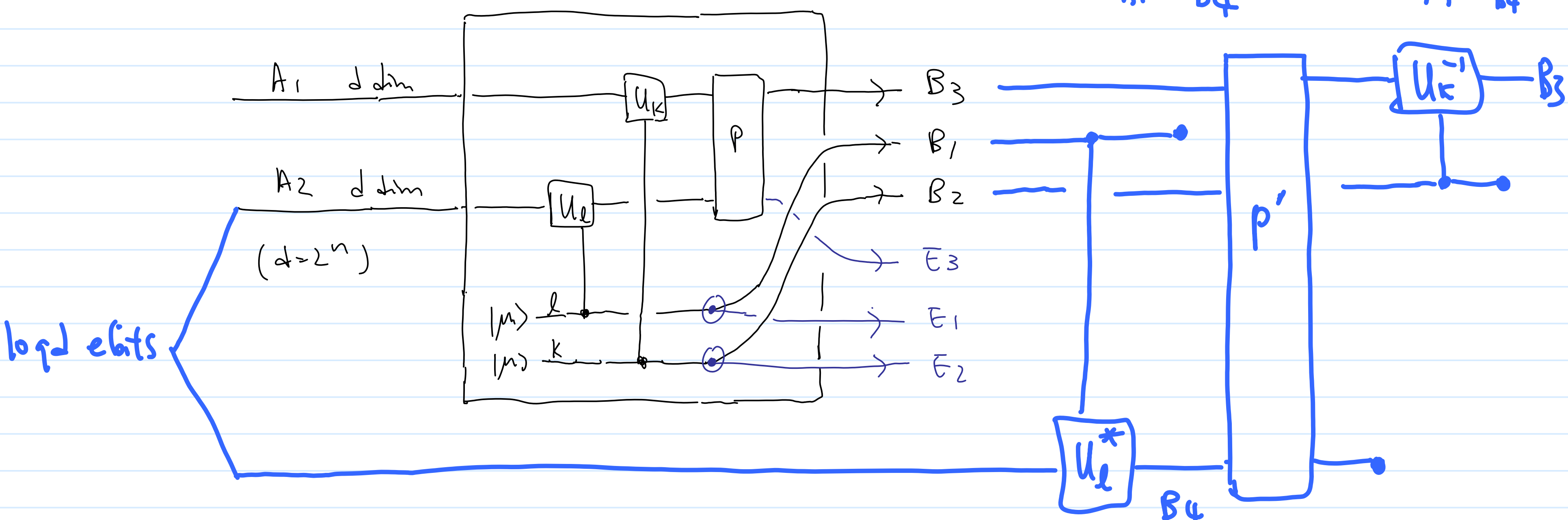
$P_E(R_d) \geq 2\log d$ ∴ $C_E(R_d) \geq 2\log d$

② $Q_E(R_d) \geq \log d$:

R_d is made to be "echo-correctable" or "retro correctable"

If A & B can consume $\log d$ ebits

$P'|i\rangle_{A_1}|j\rangle_{B_4} = \omega^{-ij}|i\rangle_{A_1}|j\rangle_{B_4}$



Then $A_1 \to \boxed{} \to B_3$ is identity channel on d dims.

∴ $Q_E(R_d) \geq \log d$,

We can take the $A_1 \to B_3$ channel & use another $\log d$ ebits to perform superdense coding.

$$\therefore \quad R_d + \log d \text{ ebit} \geq 2 \log d \text{ cobits} \rightarrow \qquad \text{— } (2a)$$

$$\therefore \quad P_E(R_d) \geq 2 \log d, \qquad C_E(R_d) \geq 2 \log d.$$

(Tempting to say "=").

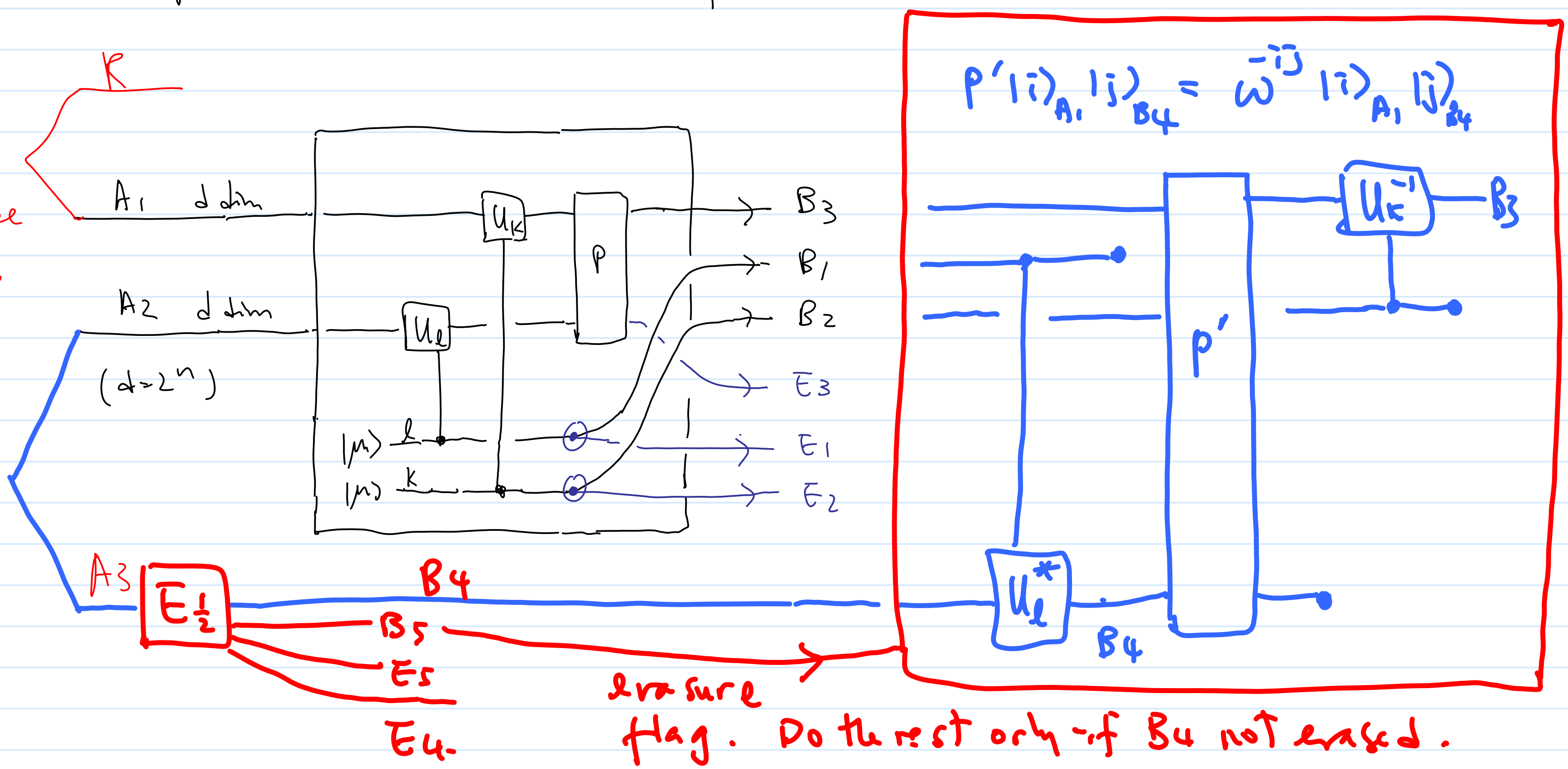
(3) $Q^{(1)}(R_d \otimes E_{\frac{1}{2}}) \geq \frac{1}{2}\log d.$

Idea: use $E_{\frac{1}{2}}$ to general $\log d$ ebits wp $\frac{1}{2}$

Then proceed w/ entanglement assisted communication.

To bound $Q^{(1)}(R_d)$

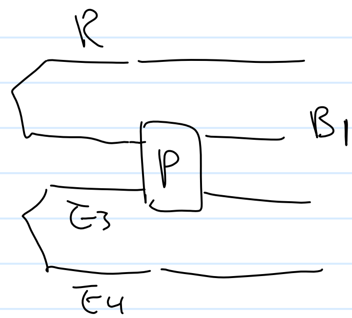
Let RA_1 be max ent.



wp $\frac{1}{2}$, B_4 not erased. $I_c(R\rangle B_1) = \log d$

wp $\frac{1}{2}$, B_4 erased, • A_2 & E_4 max ent

• for each l, K (known to Bob & Eve)



and $S(RB_1) = S(B_1) = \log d$

$\therefore I_c(R\rangle B_1) = \log d$

(in both cases, other sys with Bob irrelevant in calculating I_c.)

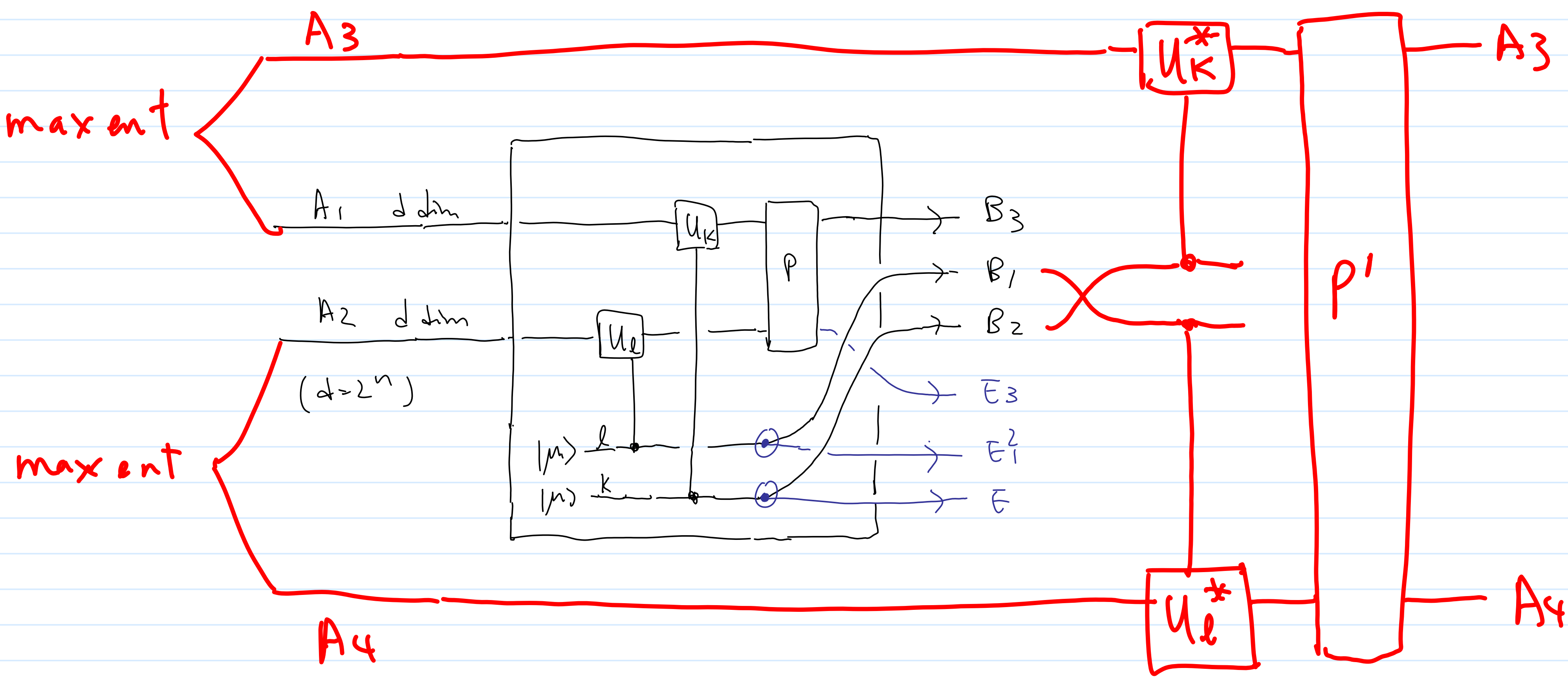
$\therefore Q^{(1)}(R_d \otimes E_{\frac{1}{2}}) \geq \frac{1}{2}\log d + \frac{1}{2}\cdot 0 = \frac{1}{2}\log d$

(Though $Q(R_d) \leq 2$, $Q(E_{\frac{1}{2}}) = 0$, very superadditive.)

$\therefore Q_{ss}(R_d) \geq \frac{1}{2}\log d$.

④ $Q_2(R_d) \geq \log d$.

Give free 2-way comm, Bob meas $B_1 B_2$ and send them to Alice. For simplicity, Bob sends $B_1 B_2$ to Alice.



So $A_3 B_3$ max ent (& $E_3 A_4$ max ent).

($R_d + 2\log|C_n|$ cbits$_\leftarrow$ $\geq \log d$ ebits) ~ 4a

Alice uses free forward CC to teleport $\log d$ qubits

($R_d + 2\log|C_n|$ cbits$_\leftarrow$ + $2\log d$ cbits$_\rightarrow$ $\geq \log d$ qbits$_\rightarrow$). — 4b

$\therefore Q_2(R_d) \geq \log d$.

If only free backward CC is available, we concatenate 4a & 2a ie generate $\log d$ ebits & run ent assisted protocol

$$\underbrace{2R_d + 2\log|C_n| \text{ cbit}_\leftarrow \geq R_d + \log d \text{ ebit}}_{4a} \underbrace{\geq \log d \text{ qbit}_\rightarrow}_{2a}$$

$\therefore Q_B \geq \frac{1}{2}\log d$.

Finally, using (4a) twice & (2a) once, we can do superdense coding:

$$3 R_d + 4 \log |C_n| \text{ cbits} \underset{(4a) \times 2}{\geq} R_d + 2 \log d \text{ ebit}$$

$$\underset{2a}{\geq} \log d \text{ qbit} + \log d \text{ ebit} \underset{SD}{\geq} 2 \log d \text{ cbit.}$$

$\therefore C_B(R_d) \geq P_B(R_d) \geq \frac{2}{3} \log d.$

$C_2(R_d) \geq \log d$ from $Q_2(R_d) \geq \log d$ but I don't see how to make C_2 larger.