

Supplement to notes for lec 1:

① References:

- Relating independence & low weight errors, def of QECC, discretization: private comm with Daniel Gottesman
- QECC criterion:
 - Ekert & Macchiavello 1996
 - Knill & Laflamme 1996
 - Bennett, DiVincenzo, Smolin, Wootters 1996
 - Nielsen, Barnum, Ceres, Schumaker 1997

Our version is due to Nielsen et al, streamlined with concepts & defs by Gottesman.

- 9-bit Shor code:
 - Shor
 - lecture notes by Chris for QIC 710

② Definition of weight and identity map if the dimension is increased by a channel:

Let $N_i: \mathcal{B}(\mathbb{C}^d) \rightarrow \mathcal{B}(\mathbb{C}^{d'})$, $d' \geq d$.

$$\text{Let } \tilde{I} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ & & & \ddots \end{bmatrix} \begin{matrix} \uparrow \\ \\ \downarrow \\ \\ \end{matrix}, \quad \tilde{I}(p) = \tilde{I} p \tilde{I}^\dagger$$

$\leftarrow d \rightarrow$

Then the weight of $A = A_1 \otimes A_2 \otimes \dots \otimes A_m$ where each $A_i: \mathbb{C}^d \rightarrow \mathbb{C}^{d'}$ can be defined as the number of A_i 's not equal to \tilde{I} .

Likewise, $\|N_i - \tilde{I}\|_2$ is a measure of how close N_i is to the identity map.

The theorems on approx indep errors by low weight error channel go through without major changes.

③ A slight simplification of P(1) of part 2, lec 1, starting from the 3rd bullet point:

• ∵ |ψ⟩, |φ⟩ lin indep,

|ψ⟩ = a₀|φ⟩ + a₁|φ⊥⟩ for some a₀, a₁ ∈ ℂ, a₁ ≠ 0, ⟨φ⊥|φ⟩ = 0.

Applying ⟨φ⊥| to the last eqn:

∫_{E, |ψ⟩} a₁ |S_{E, |ψ⟩}⟩ = ∫_{E, η(|φ⟩ + |ψ⟩)} a₁ |S_{E, η(|φ⟩ + |ψ⟩)}⟩

∵ a₁ ≠ 0, ∫_{E, |ψ⟩} = ∫_{E, η(|φ⟩ + |ψ⟩)} = ∫_{E, |φ⟩}

|S_{E, |ψ⟩}⟩ = |S_{E, η(|φ⟩ + |ψ⟩)}⟩ = |S_{E, |φ⟩}⟩

↑ by reversing the roles of |ψ⟩, |φ⟩

• ∴ ∀ |ψ⟩, ∀ E ⊂ Ω, ∫_E |ψ⟩ |S_E⟩ = ∫_E |ψ⟩ |S_E⟩

↑ ↓
indep of |ψ⟩

• To show (≠):

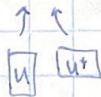
∀ |ψ⟩, |φ⟩ ∈ ℂ^k, ∀ E_i, E_j ∈ E,

(⟨φ|U†E_i†V†)(∫_{E_j} U|ψ⟩) = ∫_{E_i} ∫_{E_j} ⟨S_{E_i}|S_{E_j}⟩ ⟨φ|ψ⟩

//

⟨φ|U†E_i†E_jU|ψ⟩ = C_{ij}

Recall P = uu†, ∴ U†P = U†, PU = u, U†PU = I on ℂ^k,



⟨φ|U†PE_i†E_jPU|ψ⟩ = C_{ij} ⟨φ|U†PU|ψ⟩

∴ PE_i†E_jP = C_{ij} P