QIC890/CS867/CO781 Assignment 1

Due Friday January 21, 2020, 10:00pm

Instruction: Please submit to Crowdmark, placing the answer to each question in the right place.

Question 1. Alternative form of necessary and sufficient condition for QECC [4 marks]

You learnt from class that a codespace $\mathcal{C}$ with projector P is a QECC for the error set $\mathcal{E}$ if and only if $\forall E_i, E_j \in \mathcal{E}, PE_i^\dagger E_j P = c_{ij} P$ for some $c_{ij} \in \mathbb{C}$.

Provide a brief argument that the codespace $\mathcal{C}$ is a QECC for the error set $\mathcal{E}$ if and only if there exists an *orthonormal* basis $\{|\psi_a\rangle\}$ for $\mathcal{C}$, $\forall E_i, E_j \in \mathcal{E}, \langle\psi_a|E_i^\dagger E_j|\psi_b\rangle = c_{ij}\ \delta_{a,b}$ for some $c_{ij} \in \mathbb{C}$, and δ denotes the Kronecker delta function.

Note that we can replace "there exists an" by "for any" and obtain another equivalent statement.

Question 2. Bosonic code for amplitude damping [10 marks]

Consider an infinite-dimensional Hilbert space with a basis $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle, \cdots\}$ where $|j\rangle$ denotes a state with j excitations (e.g., j photons). Consider the amplitude damping channel $\mathcal{A}_\gamma(\rho) = \sum_k A_k \rho A_k^\dagger$ where

$$A_k = \sum_{j \geq k} \sqrt{\binom{j}{k}} \sqrt{(1-\gamma)^{j-k}\, \gamma^k}\ |j{-}k\rangle\langle j|$$

represents the loss of k excitations from the system. In particular,

$$A_0 = \sum_j (1-\gamma)^{\frac{j}{2}} |j\rangle\langle j|, \qquad A_1 = \sum_{j \geq 1} \sqrt{j\,(1-\gamma)^{j-1}\,\gamma}\ |j{-}1\rangle\langle j|\,.$$

(a) [6 marks] Show that the codespace with basis

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|40\rangle + |04\rangle)\,, \qquad |\psi_1\rangle = |22\rangle$$

is a QECC for the error set $\mathcal{E} = \{A_0 \otimes A_0, A_0 \otimes A_1, A_1 \otimes A_0\}$.

(b) [4 marks] Describe a valid decoding operation for this QECC.

Question 3. Approximate error correction [6 marks]

Consider the QECC $\mathcal{C}'$ with basis

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|4\rangle + |0\rangle)\,, \qquad |\psi_1\rangle = |2\rangle$$

and the error set $\mathcal{E}' = \{A_0, A_1\}$. Note that $\mathcal{C}'$ is *not* a QECC for $\mathcal{E}'$; provide a decoding operation $\mathcal{D}$ so that $\|\,\mathcal{D}(\mathcal{A}_\gamma(\rho)) - \rho\,\|_1 \approx O(\gamma^2)$.

PS. In the same way, any distance-3 qubit QECC also corrects for 1 amplitude damping error (requiring a blocklength of at least 5). A 4-qubit approximate QECC (in quant-ph/9704002) corrects for 1 amplitude damping error and the codespace is spanned by $\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)^{\otimes 2}$. The similar Shor code corrects for 2 amplitude damping errors (Section 8.7 of Gottesman thesis)!