QIC890/CS867/CO781 Assignment 2

Due Friday February 23, 2024, 7:00pm

Instruction: Please submit to Crowdmark, placing the answer to each question in the right place.

Question 1. Unitary conjugating the generators for the Pauli group in specific ways [5 marks]

Please derive a unitary U satisfying the following:

$$UXIIU^\dagger = ZII$$
$$UZIIU^\dagger = XZI$$
$$UIXIU^\dagger = ZXX$$
$$UIZIU^\dagger = IZI$$
$$UIIXU^\dagger = IZZ$$
$$UIIZU^\dagger = IIX$$

using the method described in lecture 4. Please present your derivation in a way that facilitates verification, and present the unitary U as an 8×8 complex unitary matrix

Question 2. Encoded Clifford operation for the 5-qubit code? [5 marks]

Let U denote the single qubit Clifford gate that achieves the following conjugation map:

$$UXU^\dagger = Y \tag{1}$$
$$UZU^\dagger = X \tag{2}$$

Note that $UYU^\dagger = U(iXZ)U^\dagger = iUXU^\dagger UZU^\dagger = iYX = Z$.

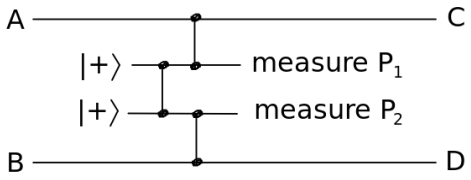
Recall the 5-qubit code has stabilizer S and encoded Pauli's with the following generators:

$$\begin{array}{rcccccc} Q_1 & = & X & Z & Z & X & I \\ Q_2 & = & I & X & Z & Z & X \\ Q_3 & = & X & I & X & Z & Z \\ Q_4 & = & Z & X & I & X & Z \\ \bar{X} & = & X & X & X & X & X \\ \bar{Z} & = & Z & Z & Z & Z & Z \end{array}$$

Is $U^{\otimes 5}$ an encoded operator on the 5-qubit code? Provide a proof for your answer. If the answer is yes, what encoded Clifford gate is being implemented by $U^{\otimes 5}$?

Question 3. Tracking evolution in the stabilizer formalism [10 marks]

Consider the following circuit, where the vertical line ending with two dots represent a controlled-Z gate, and the systems A and B contain an *arbitrary* 2-qubit input state,



Track the evolution of the state in the stabilizer framework, for
(a) [4 marks] $P_1 = P_2 = X$.
(b) [3 marks] $P_1 = P_2 = Z$.
Note that in each case, we make 2 measurements (e.g., in part (a) we measure $IXII$ and then $IIXI$ with 4 outcomes, rather than measuring $IXXI$ with 2 outcomes).

(c) [3 marks] By choosing between these 2 possible measurement schemes in parts (a) and (b), what can the circuit accomplish?