QIC890/CS867/CO781 Assignment 3

Due Wednesday February 09, 2022, 5:00pm

Instruction: Please submit to Crowdmark, placing the answer to each question in the right place.

Question 1. Given the images of conjugating the generators for the Pauli group ... [8 marks]

Please derive the symplectic representation $\mathbb{S}(U)$ (6×6 binary matrix), the unitary U (8×8 complex unitary matrix), and the circuit for U in terms of simple gates (Pauli's, $R_{\pm\pi/4}$, Hadamard, SWAP, CNOT, controlled-Z's), given the following:

$$
\begin{aligned}
UXIIU^\dagger &= ZII \\
UZIIU^\dagger &= XZI \\
UIXIU^\dagger &= ZXX \\
UIZIU^\dagger &= IZI \\
UIIXU^\dagger &= IZZ \\
UIIZU^\dagger &= IIX
\end{aligned}
$$

This is an exercise to gain familiarity of the course material. There are multiple approaches and you are free to take any. You can use numerics (but this may not be needed). There can be different circuits implementing the same unitary. Please present your answers in a way which facilitates verification.

Question 2. Encoded Clifford operation for the 5-qubit code? [6 marks]

Let U denote the single qubit Clifford gate that achieves the following conjugation map:

$$UXU^\dagger = Y \tag{1}$$

$$UZU^\dagger = X \tag{2}$$

Note that $UYU^\dagger = U(iXZ)U^\dagger = iUXU^\dagger UZU^\dagger = iYX = Z$.

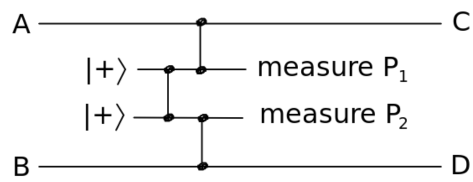
Recall the 5-qubit code has stabilizer S and encoded Pauli's with the following generators:

$$
\begin{array}{ccccccc}
Q_1 & = & X & Z & Z & X & I \\
Q_2 & = & I & X & Z & Z & X \\
Q_3 & = & X & I & X & Z & Z \\
Q_4 & = & Z & X & I & X & Z \\
\bar{X} & = & X & X & X & X & X \\
\bar{Z} & = & Z & Z & Z & Z & Z
\end{array}
$$

Is $U^{\otimes 5}$ an encoded operator on the 5-qubit code? Provide a proof for your answer. If the answer is yes, what encoded Clifford gate is being implemented by $U^{\otimes 5}$?

Question 3. Tracking evolution in the stabilizer formalism [8 marks]

Consider the following circuit, where the vertical line ending with two dots represent a controlled-Z gate, and the systems A and B contain an *arbitrary* 2-qubit input state,



Track the evolution of the state in the stabilizer framework, for
(a) [3 marks] $P_1 = P_2 = X$.
(b) [3 marks] $P_1 = P_2 = Z$.
Note that in each case, we make 2 measurements (e.g., in part (a) we measure $IXII$ and then $IIXI$ with 4 outcomes, rather than measuring $IXXI$ with 2 outcomes).

(c) [2 marks] By choosing between these 2 possible measurement schemes in part (a) and (b), what does the circuit accomplish?