

Fall 2020 QIC 890 / CO 781 Assignment 4

Due Monday Dec 07, 2020 on Crowdmark

Question 1. The quantum capacity of the amplitude damping channel

Recall that the amplitude damping channel \mathcal{N}_γ has a Kraus representation

$$\mathcal{N}(\rho) = A_0\rho A_0^\dagger + A_1\rho A_1^\dagger \text{ where } A_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} \text{ and } A_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}.$$

(a) [3 marks] Find an isometric extension U_γ for \mathcal{N}_γ . You can give a circuit, or specify the image of an arbitrary input. In the latter, justify why the mapping is isometric. In either case, explain very briefly why the mapping is an extension for \mathcal{N}_γ .

(b) [1 mark] What is \mathcal{N}_γ^c ?

(c) [3 marks] What is $\mathcal{N}_\eta \circ \mathcal{N}_\gamma$?

(d) [3 marks] Show that \mathcal{N}_γ is degradable for a certain range of γ . Show what is this range, and give a degrading map and show that it “degrades”.

(e) [5 marks] Find the optimal state for the 1-shot coherent information for \mathcal{N}_γ and evaluate $Q^{(1)}(\mathcal{N}_\gamma)$. Consider cases $\gamma \geq 1/2$ and $\gamma \leq 1/2$ separately.

You can assume without proof that, if \mathcal{N} is degradable, $|\phi_i\rangle$ purifies ρ_i for $i = 0, 1$, then,

$$pI_c(R)B)_{I \otimes \mathcal{N}(|\phi_0\rangle\langle\phi_0|)} + (1-p)I_c(R)B)_{I \otimes \mathcal{N}(|\phi_1\rangle\langle\phi_1|)} \leq I_c(R)B)_{I \otimes \mathcal{N}(|\phi\rangle\langle\phi|)}$$

where $|\phi\rangle$ purifies $p\rho_0 + (1-p)\rho_1$. Use this fact to show the optimal state has the form $(\sqrt{1-\beta}|00\rangle + \sqrt{\beta}|11\rangle)_{RA}$. You can leave the answer as an optimization over β for a simple enough function, and plot the optimal β as a function of γ .

(f) [1 mark] Find $Q(\mathcal{N}_\gamma)$.

Question 2. Question removed to give more time for term project [4 marks]

Each student will automatically receive 4 marks. For this to work with Crowdmark, please submit an empty page (so I can enter the marks).