Quantum Error Correction and Fault Tolerance, Winter 2022

Part II: Fault Tolerance

Problem Set 1

Deadlines

- Q1 and Q2 due on Friday February 18 2022 10pm.
- Q3 (EC Zoo) 1st round due on Friday February 25 2022 10pm.

1. Flag error correction [8 marks]

(a) The 5-qubit code has stabilizer by $XZZXI$ and its cyclic permutations. Its logical operators are $\overline{X} = XXXXX$ and $\overline{Z} = ZZZZZ$. The naive circuit for measuring the stabilizer $XZZXI$ is shown in Figure 1. Assume that the first CZ is followed by one of XI, IX, XX, ZI, IZ ZZ, IY, YI, YY, XZ, ZX, XY, YX, ZY, YZ, chosen at random, acting on the two qubits in the support of the first CZ. List all of the 'bad' faults that can lead to a multi-qubit data error, and their corresponding data error. [**4 marks**]

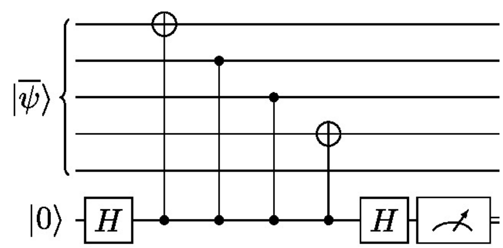

Figure 1: Naive 5-qubit code stabilizer measurement circuit

(b) The circuit shown in Figure 2 incorporates a flag qubit to catch the bad faults. For each bad fault identified above, verify that the measurement of the flag qubit yields -1, heralding the bad fault. [**4 marks**]

2. Steane error correction [12 marks]

Suppose we encode a qubit in a $k = 1$ CSS code. In Steane error correction we use logical $|\overline{0}\rangle$ and $|\overline{+}\rangle$ ancillas, encoded in the same code as the data. We can extract all the stabilizer eigenvalues simultaneously using the circuit in Figure 3. The X stabilizers of a CSS code define a classical code. To find the recovery operator in Steane error correction, we simply do classical error correction on the ancilla measurement outcomes.

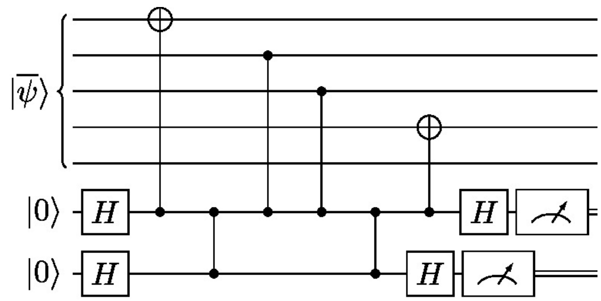

Figure 2: Flag 5-qubit code stabilizer measurement circuit

(a) Recall that Steane's code is a [[7,1,3]] code with stabilizer generators $X_1X_2X_4X_5$, $X_2X_3X_4X_6$, $X_4X_5X_6X_7$, $Z_1Z_2Z_4Z_5$, $Z_2Z_3Z_4Z_6$, $Z_4Z_5Z_6Z_7$, and logical operators $\overline{X} = X^{\otimes 7}$ and $\overline{Z} = Z^{\otimes 7}$. Write down the logical $|\overline{0}\rangle$ and $|\overline{+}\rangle$ states for this code. **[2 marks]**

(b) Suppose there is an X_1Z_4 error on the data. Apply Steane error correction and write down the error syndrome (the list of stabilizer generators with -1 eigenvalue). What is the minimum weight recovery operator? **[3 marks]**

(c) Suppose there is an X_3X_4 error on the data and an X_1Z_7 error on the $|\overline{+}\rangle$ ancilla. Apply Steane error correction and compute the error syndrome and the minimum weight recovery operator. **[2 marks]**

(d) Let p be the probability of any fault in the Steane stabilizer measurement circuit. Assume a depolarizing noise model, i.e. each single-qubit gate is replaced by the gate followed by the depolarizing channel $\mathcal{E}(\rho) = (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z)$. Similarly, there is a depolarizing channel before each measurement. And each two-qubit gate is replaced by the ideal gate followed by a two-qubit depolaring channel, i.e. with probability $(1-p)$ apply II, and with probability p apply one of XI, IX, XX, ZI, IZ ZZ, IY, YI, YY, ZX, XZ, XY, YX, ZY, YZ, chosen at random.
Prove that Steane error correction succeeds with probability $1 - O(p^2)$. Success here means that performing ideal error correction on the output state state gives the same result as applying ideal error correction on the input state. You can assume that recovery operators are implemented perfectly (in real processors they are tracked in software). **[5 marks]**

Hint: You need to show that Steane error correction succeeds for each error occurring with probability p.

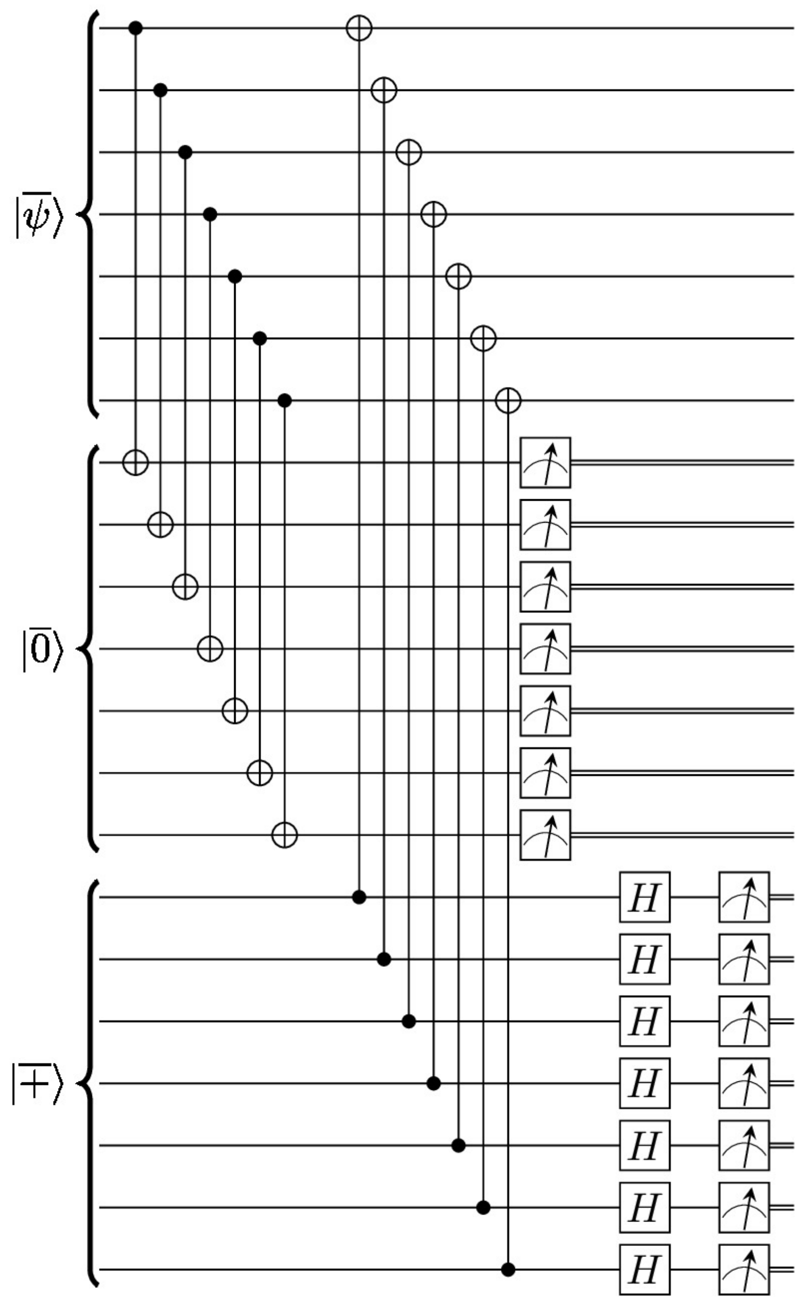

Figure 3: Steane error correction circuit

3. Error correction zoo [10 marks]

Goal: To provide an encyclopedia of error correcting codes for classical and quantum coding theory communities. EC Zoo website.

- Everything organized in terms of lists of code families (e.g. LDPC codes, CV codes, binary codes, codes with transversal gates).
- Related: complexity and algorithm zoos.

Your job: contribute several code family entries.

Two birds / one stone: Use information collected for final project for the zoo, and visa versa.

How to pick codes: Pick a code not in the zoo using this spreadsheet or propose one yourself. Pick only codes with difficulty 1-3, adding up 4 total; can propose other codes.

- No more than 2 points come from classical codes.
- At least one code should be level >1.

Add codes to your name in the "2022 IQC QEC" sheet and strikethrough code names once added.

As you research: If you see more codes while reading papers, please either add/modify entries (for extra credit), or add codes to the spreadsheet. Work together on similar entry groups, communicate, discuss parents/cousins; on Github, all contributions will be counted. For codes of difficulty 3, spin-off entries may be necessary. Spend some time looking up connections to other codes. Lack of papers in the spreadsheet doesn't mean I don't know what you should be including.

How to submit:

1. Go to this github repo.
2. Download template.yml file, change it according to code you picked, and remove all instructional comments.
3. Navigate to folder where code should lie and add/replace code file (you need an account to do this). The format for template.yml file may change (please bear with us). Submit one commit with all the code files you were assigned.
4. There will be at least one round of review and resubmission. Once I make comments on Round 1, please address them, resolve any Github conversations, and submit a second "Round 2" commit with all your assigned codes and the comments addressed.
5. Make sure that Round 2 passes Github checks. If necessary, comment out parents/cousins not in the zoo in order to make checks pass, or make additional stub entries for them.