(Q1) Consider $l \geq 3$ orthogonal pure states $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_l\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^n$,

where $n \geq 2$.

For simplicity, we use vectors $|\psi_i\rangle$ instead of $|\psi_i\rangle\langle\psi_i|$ for states.

Let Alice has the system corresponding to $\mathbb{C}^2$

.. Bob . . . $\mathbb{C}^n$

and let Alice make the first nontrivial measurement;

ie $M_0 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ for some $1 \geq a \underset{\neq}{\geq} b \geq 0$

in some basis.

There are no other restriction to the LOCC protocol (# rounds, # meas outcomes etc).

Show that the l states can be perfectly discriminated

$\Leftrightarrow$ $\exists$ basis for Alice $\{|0\rangle, |1\rangle\}$

s.t $\forall i = 1, \ldots, l$, $|\psi_i\rangle = |0\rangle \otimes |\beta_0^i\rangle + |1\rangle \otimes |\beta_1^i\rangle$

and $\forall i \neq j$, $\langle \beta_0^i | \beta_0^j \rangle = 0$, $\langle \beta_1^i | \beta_1^j \rangle = 0$

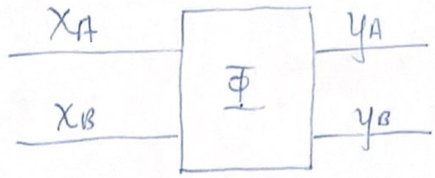
[Note that $|\beta_0^i\rangle, |\beta_1^i\rangle$ are subnormalized vectors

and can even be zero, but $|\psi_i\rangle$ is a unit vector.]

- Note also that for perfect discrimination of a finite set, the distribution of the states are irrelevant, assuming all $p_i > 0$.

- For the direction $[\Leftarrow]$, clearly describe the protocol (proof of correctness not needed). The direction $[\Rightarrow]$ carries 4 marks; it is not difficult, but you need to fill in the few major conceptual steps besides the correct mathematics.

(Q2) Consider the CP maps Φ taking $L(X_A \otimes X_B)$ to $L(Y_A \otimes Y_B)$.



Show that the following 3 conditions on Φ are equivalent, where T_S denotes transpose map on $L(S)$ and identity map elsewhere.

(a) $T_{Y_A} \circ \Phi \circ T_{X_A}$ is a CP map taking $L(X_A \otimes X_B)$ to $L(Y_A \otimes Y_B)$

(b) $J(\Phi) = \Phi \otimes I_{\tilde{X}_A \tilde{Y}_A} (\beta_A \beta_A^* \otimes \beta_B \beta_B^*) \in PPT(\tilde{X}_A Y_A : \tilde{X}_B Y_B)$

$$\beta_A = \sum_{i=1}^{\dim(X_A)} |i\rangle_{\tilde{X}_A} \otimes |i\rangle_{X_A}$$

$$\beta_B = \sum_{j=1}^{\dim(X_B)} |j\rangle_{\tilde{X}_B} \otimes |i\rangle_{X_B}$$

(c) $\forall\, P \in PPT(\tilde{X}_A X_A : \tilde{X}_B X_B)$

$I_{\tilde{X}_A \tilde{X}_B} \otimes \Phi(P) \in PPT(\tilde{X}_A Y_A : \tilde{X}_B Y_B)$

- Historically, (a) was first proposed by Eric Rains, and equivalence to (c) was confirmed by him later.

NB: (b) makes it obvious SEP $\subseteq$ PPT.

- Suggestion: with mark allocation

$$\begin{tikzcd} (a) \arrow[r, Leftrightarrow, "\text{easy (4)}"'] \arrow[rr, bend left, "\text{not difficult (4)}"] & (b) & (c) \arrow[l, Rightarrow, "\text{easy (1)}"] \end{tikzcd}$$

Warning: (b) $\Rightarrow$ (c) via the last eqn of Sec 5.2.2 in LN 2011 possible, but requires several identities concerning partial transpose and partial trace.