Fall 2016 QIC 890 / CO 781 Assignment 4

Due Dec 01, 2016 (in class)

Question 1. The quantum capacity of the amplitude damping channel

Recall that the amplitude damping channel $\mathcal{N}_\gamma$ has a Kraus representation

$$\mathcal{N}(\rho) = A_0 \rho A_0^\dagger + A_1 \rho A_1^\dagger \text{ where } A_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} \text{ and } A_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}.$$

(a) Find an isometric extension U_γ for $\mathcal{N}_\gamma$. You can give a circuit, or specify the image of an arbitrary input. In the latter, justify why the mapping is isometric. In either case, explain very briefly why the mapping extends $\mathcal{N}_\gamma$.

(b) What is $\mathcal{N}_\gamma^c$?

(c) What is $\mathcal{N}_\eta \circ \mathcal{N}_\gamma$?

(d) Show that $\mathcal{N}_\gamma$ is degradable for a certain range of γ. Show what is this range, and give a degrading map and show that it "degrades".

(e) Find the optimal state for the 1-shot coherent information for $\mathcal{N}_\gamma$) and evaluate $Q^{(1)}(\mathcal{N}_\gamma)$. Consider cases $\gamma \geq 1/2$ and $\gamma \leq 1/2$ separately.

You can assume without proof that, if $\mathcal{N}$ is degradable, $|\phi_i\rangle$ purifies ρ_i for $i = 0, 1$, then,

$$p I_c(R\rangle B)_{I\otimes\mathcal{N}(|\phi_0\rangle\langle\phi_0|)} + (1-p) I_c(R\rangle B)_{I\otimes\mathcal{N}(|\phi_1\rangle\langle\phi_1|)} \leq I_c(R\rangle B)_{I\otimes\mathcal{N}(|\phi\rangle\langle\phi|)}$$

where $|\phi\rangle$ purifies $p\rho_0 + (1-p)\rho_1$. Use this fact to show the optimal state has the form $(\sqrt{1-\beta}|00\rangle + \sqrt{\beta}|11\rangle)_{RA}$. You can leave the answer as an optimization over β for a simple enough function, and plot the optimal β as a function of γ.

(f) Find $Q(\mathcal{N}_\gamma)$.

Question 2. Upper and lower bounds for the quantum capacity of a random Pauli channel

Let $\mathcal{N}(\rho) = 0.8\rho + 0.15 X\rho X + 0.04 Y\rho Y + 0.01 Z\rho Z$.

Using the method of additive/degradable extension to upper bound $Q(\mathcal{N})$.

Compare the above to $Q^{(1)}(\mathcal{N})$ (assuming the optimal state is maximally entangled between the reference R and the input).