(3) Beyond QM?

The 5 axioms of QM provide a for a physical world without much pathologies.

What are pathologies?
- grand father paradox in time travel
- super luminal communications etc.

A plethora of interpretations of QM all give the same prediction.

For example: sys A is in a state given by ρ

vs sys AR is in a pure state $|\psi\rangle_{AR}$, $\rho = \mathrm{tr}_R |\psi\rangle\langle\psi|$.

There has been a significant body of work concerning the "computational / information theoretic advantages" if QM is "augmented".

eg. Post BQP allows post selection of measurement outcome

$BQP_{D\text{-}CTC}$ allows Deutsch closed-time-like-curves to be used as black boxes.

Allows cloning or beyond-QM ability to discriminate Q states.

Warning:

Most results in this direct assume features / interpretations of QM that may not be consistent with the extension.

The simplest model to analyse : Post BQP.

Idea : allows measurements along $\{|0\rangle, |1\rangle\}$

post select "$|0\rangle$", renormalize remaining states.

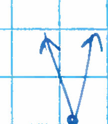
Aaronson 04 : Post BQP = PP.

= class of decision problems solvable by a probabilistic TM in poly time with prob of error $< \frac{1}{2}$.

Wide belief : PP >> BPP.

* Surprisingly, these measurements can be delayed until the last step in the computation and usual QM be applied except the measurement step.

* Can post select in unambiguous state discrimination problems (either identify the state correctly or say I don't know) and identify 2 close-by pure states perfectly.



Problems:

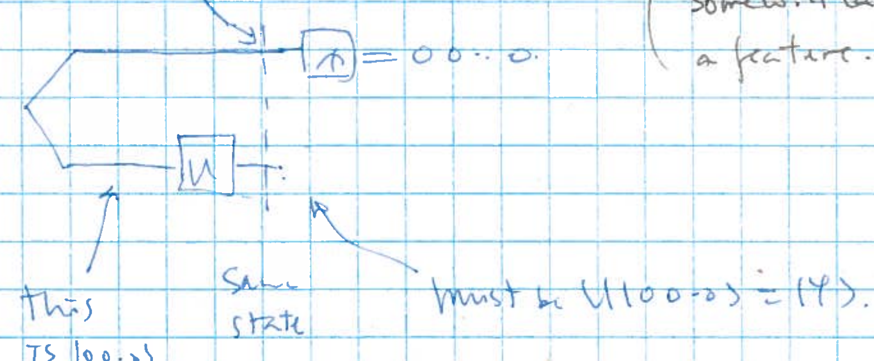
- 1 postselected measurement allows communication of a 2-dim pure state from Alice to Bob.
  n such measurements communicates a 2^n-dim pure state $|\psi\rangle$.

Method: Complete $|\psi\rangle$ to a basis: $\{|\psi\rangle = |\psi_0\rangle, |\psi_1\rangle, \ldots, |\psi_{2^n-1}\rangle\}$

Let $|\psi_i\rangle = U|\text{binary}(i)\rangle$

Alice applies U^T on A, then post select "$|0\cdots0\rangle$".

By the transpose trick,



Superluminal communication!

(Some will call it a feature....)

Question: Bob receives no message, does nothing.

By Alice's action alone, state in B changes from $\frac{I}{2^n}$ to $|\psi\rangle\langle\psi|$.

Does he still have a state (a density matrix) he can describe his system with?

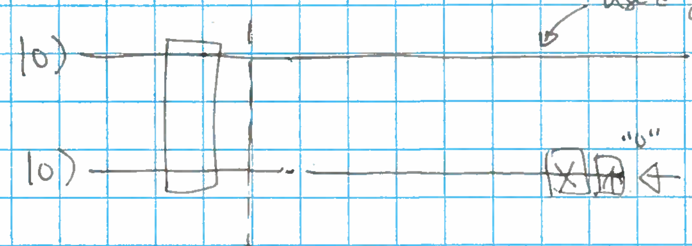
What if Alice cheats and skip her operations?
What is Bob's state?!

In the least we cannot describe a quantum system by a local density matrix.

- Similarly, TP can be done WITHOUT 2cbits but 2 such post-selected measurements.

- Furthermore, an adversary can alter a state "back in time":

used for a computation etc



Sudden Eve flips her qubit and post select "|0⟩". Effectively flipping a "|0⟩" in the lab to a "|1⟩" that's already used in the circuit.

$(1-\varepsilon)\,|0\rangle\langle 0|^{\otimes 2} + \varepsilon\,|1\rangle\langle 1|^{\otimes 2}$

- Do we still have states?
- Do we have "space" (lost of locality)?
- Does it make sense to talk about computation & complexity classes?

Example: cloning + QM.

This is MUCH TRICKIER.

Linearity of QM ⇒ no cloning.

So necessarily we give up linearity to accept a magic cloning machine.

Say: Bob is given a cloning machine.

What does it mean?

A mapping $\mathcal{E}(\psi) = \psi^{\otimes 2}$?

Now what is state being cloned?

If Bob knows the state he just makes more copies.

So the state is determined outside of Bob's Control.

e.g. $\sum_x p_x \underbrace{|x\rangle\langle x|}_{\substack{\text{the label of} \\ \text{the unknown} \\ \text{state not} \\ \text{accessible to Bob}}} \otimes \underbrace{|\psi_x\rangle\langle\psi_x|}_{\substack{\text{the state} \\ \text{unknown} \\ \text{to Bob}}}$

ie this describes an ensemble of states, $|\psi_x\rangle$ occurs with prob p_x.

Successful cloning means

$$\sum_x p_x |x\rangle\langle x| \otimes |\psi_x\rangle\langle\psi_x| \longrightarrow \sum_x p_x |x\rangle\langle x| \otimes |\psi_x\rangle\langle\psi_x|^{\otimes 2}$$

If $\mathcal{E}(\psi) = \psi^{\otimes 2}$,

What is $I \otimes \mathcal{E}\left(\sum_x p_x |x\rangle\langle x| \otimes |\psi_x\rangle\langle\psi_x|\right)$?

We don't have linearity !!

Option 1: The "input" to $\mathcal{E}$ is $\sum_x p_x |\psi_x\rangle\langle\psi_x|$!

But Bob knows this state. He can prepare 2 copies.

Cloning machine is useless.

(I think this is a prefered option.).

Option 2: The cloner sees the state $|\psi_x\rangle\langle\psi_x|$ if the other sys is $|x\rangle\langle x|$.

Qn: What if $\mathcal{E}$ is applied to half of a max ent state?

(Warning: Choi-rep is broken!).

$\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) = \frac{1}{\sqrt{2}}(|{+}{+}\rangle+|{-}{-}\rangle) = |\Phi\rangle$

Should $I \otimes \mathcal{E}(|\Phi\rangle\langle\Phi|) = \frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|)$

or $\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$?

or $\frac{1}{2}(|{+}{+}{+}\rangle\langle{+}{+}{+}| + |{-}{-}{-}\rangle\langle{-}{-}{-}|)$?

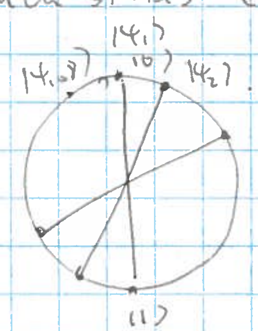
or $\frac{1}{\sqrt{2}}(|{+}{+}{+}\rangle + |{-}{-}{-}\rangle)$?

OK. insist some interpretation of QM such that "cloning works".

Then we again run into superluminal communication:

Alice decides on $|\psi\rangle = a|0\rangle + b|1\rangle$, $a, b \in \mathbb{R}$.

Consider 108 such states evenly spread on the x-z plane



Alice & Bob share 1 EPR pair, and perform RSP to transmit $|\psi\rangle$ to Bob by using 1 cbit.

Their state : $\sum_{x=1}^{108} p_x |x\rangle\langle x| \otimes |\psi_x\rangle\langle\psi_x|$

$p_x = \frac{1}{108}$

If cloning machine is successful, Bob can convert state to

$$\sum_{x=1}^{108} \frac{1}{108} |x\rangle\langle x| \otimes |\psi_x\rangle\langle\psi_x|^{\otimes 2} \leftarrow \text{or } r \text{ if}$$

Bob can use cloning machine r-1 times

Make r large enough so $|\psi_x\rangle^{\otimes r}$ are easily discriminated.

So Bob learns $\approx \log 108$ bits

Now follow the proof for (C2) and Bob guess that 1 bit.

With prob $\frac{1}{2}$, he gets one out of 108 symbols correctly

$\Rightarrow$ superluminal communication

Example: Deutsch CTC.

Similar linearity problems.

Claims to be able to clone, or discriminate $|0\rangle, |1\rangle, |+\rangle, |-\rangle$

relies on interpretations of QM that leads to

violation of Lorentz covariance etc.