Assisted capacities:

- The HSW & LSD Thms give the <u>unassisted</u> classical & quantum capacities of a quantum channel N, which is the only nonlocal resource available.
- We can consider assisted capacities instead, meaning that some extra unlimited nonlocal resource is available for free.

eg Q_E = quantum, C_E = classical — capacity of N given free entanglement.

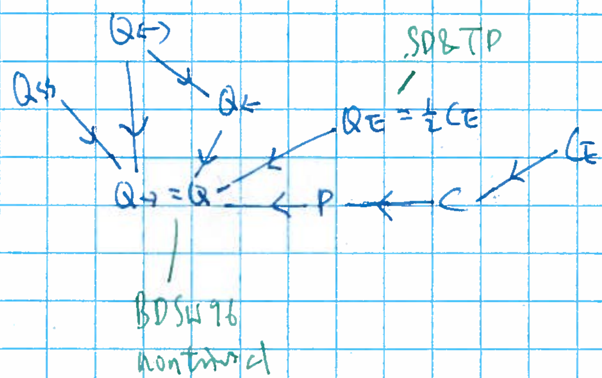
Q_{ss}, (Q_1) $Q_\rightarrow$, (Q_B) $Q_\leftarrow$, (Q_2) $Q_\leftrightarrow$ — Quantum capacity of N given free:

- side channel symmetric wrt B & E
- forward classical channel
- backward " "
- 2 way " "

- Assisting resources shouldn't trivialize the task:
  eg symmetric channels, entanglement, classical communication each cannot result in q. communication, so you're still seeing what N is capable of.

- Motivations:
  - operational interest (eg $Q_\leftrightarrow(N)$)
  - Simplified upper bounds (eg Q_{ss}, C_E both single-letter!)
  - Lends intuitions & inspires unexpected results (superactivation)

Some known relations between capacities:



Here, $\to$ means $\geq$ for all channels.

In all $\to$'s above, $\exists N$ separating the 2 capacities.

Entanglement assisted classical capacity:

Bennett Shor Smolin Thapliyal 01 06 052

$$C_E(N) = \max_{|\psi\rangle_{RA}} S(R:B)_{I\otimes N(|\psi\rangle\langle\psi|)}$$

compared to

$$\underset{\text{classical}}{C}\,(\underset{\text{quantum}}{N}) = \sup_n \frac{1}{n} \max_{\Lambda = \sum_x p_x |x\rangle\langle x|_R \otimes \rho_{x_A}} S(R:B)_{I\otimes N^{\otimes n}(\Lambda)}$$

$B_1 \ldots B_n$

$A_1 \ldots A_n$

regularized

or

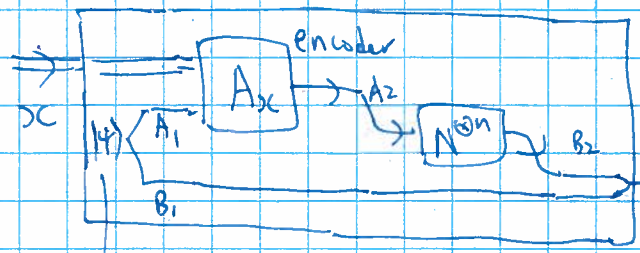
$$\underset{\text{classical}}{C}\,(\underset{\text{classical}}{N}) = \max_{\Lambda = \sum_x p_x |x\rangle\langle x|_R \otimes |x\rangle\langle x|_A} S(R:B)_{I\otimes N(\Lambda)}$$

single-letter

Hamoon will present on $C_E(N)$ Dec 6.

I'll only say 3 things here...

① Most general protocol with n-uses gives a Q-box:



$$\left(I \otimes N^{\otimes n} A_x\right)\left(|\psi\rangle\langle\psi|\right) = \rho^x_{B_1 B_2}$$

pre-stored entangled state independent of message m

② The capacity expression for which we can prove a direct coding theorem & a converse turns out ADDITIVE!

③ The direct coding theorem requires only $|\psi\rangle$ = max ent states and A_x unitary ....

④ If N classical, $C_E = C$ so entanglement does not increase the classical capacity of classical channels.

Maria Sobchuk will present how entanglement can help if comm is ZERO ERROR!

④ (Quantum) Reverse Shannon Theorem

Consider classical channels.

identity channel

- Direct coding theorem : $\boxed{nN \geq nC(N) \cdot I}$ for large n

approx simulation

NB: Happens that shared randomness, back communication etc does not increase capacity.

(Converse unchanged & the capacity achieving rate does not need any of them.)

- The reverse Shannon theorem (BSST01) says:

$$\boxed{\begin{array}{l} nC(N) \cdot I \\ + \infty \text{ rbits} \end{array} \geq nN}$$ for large n.

Brilliant

Why such a perverted idea?

Because every classical channel can be described by only 1 parameter which is $C(N)$!

Reason: $n_1 N_1 \geq n_1 C(N_1) I \geq n_1 C(N_1) \times \frac{N_2}{C(N_2)}$

any   ST   RST

and each simulation is reversible $\leq$ RST $\leq$ ST

So all channel quantities equivalent asymptotically up to this conversion rate.

So any N_1 convertible to N_2 at a rate $\frac{C(N_1)}{C(N_2)}$.

There is NO CHANCE for such a simple picture to hold for quantum channel (as manifest on Thur).

But, if entanglement is free, the following resource inequality is possible:

QRST 0912.5337 Bennett Devetak Harrow Shor Winter (2000)

$n\,C_E(N)$ cbits + $\underline{\text{arbitrary}}$ entanglt $\geq$ $n\,N$

Alt pf 0912.3805 Berta, Christandl, Renner

$n\,C_E(N)$ cbits $\underset{\text{BSST}}{\leq}$ $n\,N$ + ∞ ebits

For detail, see guest lectures by Will Matthews in S2010 offering.

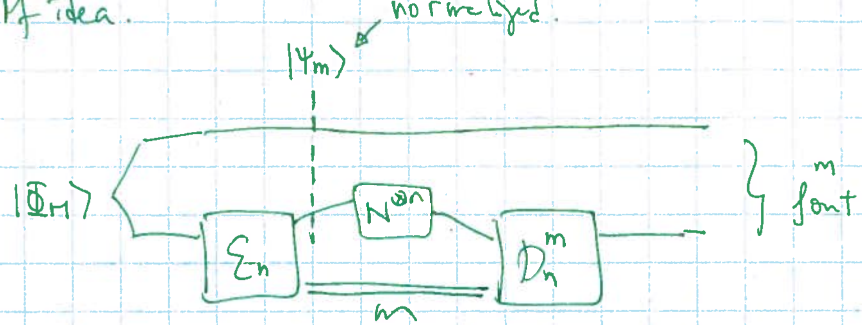
embezzlement state needed.

$\underline{\text{Thm 1}}$ $\forall N$, $Q(N) = Q_{\to}(N)$.

ie free forward CC does not increase ASYMPTOTIC Q capacity.

Pf idea.

Idea: at least some outcomes are good & just "hard wire" one of them.



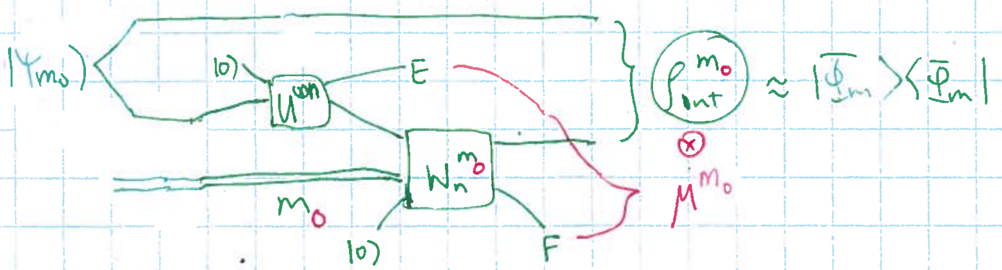
Output $\rho_{out} = \sum_m p_m \rho_{out}^m \approx |\Phi_M\rangle\langle\Phi_M|$

hardwire the good outcome.

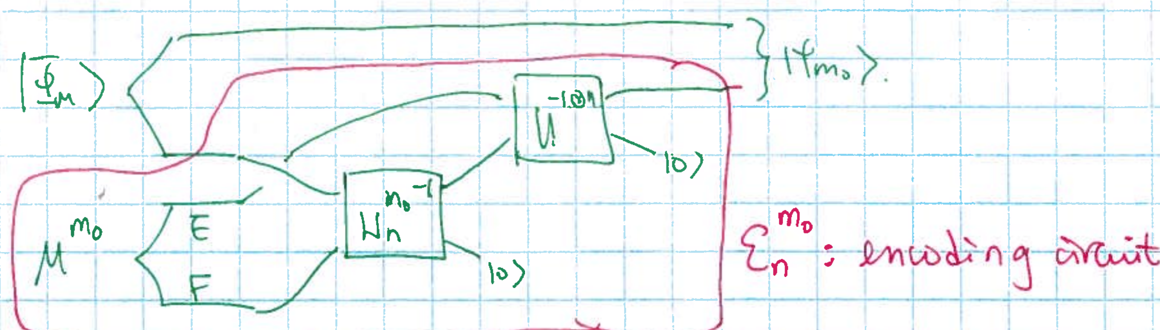
$\therefore \exists m_0$ s.t. $\rho_{out}^{m_0} \approx |\Phi_M\rangle\langle\Phi_M|$. If $\exists\ \mathcal{E}_n^{m_0}$ s.t.

then we're done. (Since m_0 need not be comm anymore.)

What is $|\psi_{m_0}\rangle$?



NB. U, W_n^m iso exts of N & $\mathcal{D}_n^m$.



For the erasure channel, say with prob of erasure $p \geq \frac{1}{2}$

$Q(N_p) = 0$

$Q_{\leftarrow}(N_p) \geq \frac{1-p}{3}$

$Q_{\leftrightarrow}(N_p) = 1-p$.

Open problem:

(or a rate region of pairs of forward & backward comm rates.)

What is the capacity (from Alice to Bob)

if N_p can communicate in either direction

and the total # of uses is charged?

This also relates to noisy channels used in

communication complexity (see Mária Kieferova's presentation).