**SPECIAL K**
**Saturday October 27, 2001**
**9:00 am - 12:00 noon**

**1:** Prove that $\sum \frac{1}{i_1 i_2 \cdots i_k} = 2001$, where the summation taken is over all non-empty subsets $\{i_1, i_2, \cdots, i_k\}$ of the set $\{1, 2, \cdots, 2001\}$.

**2:** A diameter $AB$ of a circle intersects a chord $CD$ at the point $E$. If $CE = 7$, $DE = 1$ and $\angle BED = \frac{\pi}{4}$, then determine the radius of the circle.

**3:** For any positive integer $k$, consider the sequence $a_n = \sqrt{k + \sqrt{k + \cdots + \sqrt{k}}}$, where there are $n$ square root signs.

(a) Show that the sequence $\{a_n\}$ converges for every fixed integer $k \geq 1$.

(b) Find all integers $k$ such that the limit is an integer.

(c) Show that if $k$ is odd, then the limit is irrational.

**4:** Determine all functions $f : \mathbf{R}^+ \to \mathbf{R}$ such that $f(x + y) = f(x^2 + y^2)$ for all $x, y \in \mathbf{R}^+$, where $\mathbf{R}^+$ is the set of all positive real numbers.

**5:** Find all pairs $(m, n)$ of positive integers such that $\gcd\big((n+1)^m - n, (n+1)^{m+3} - n\big) > 1$.

**BIG E**
**Saturday October, 2001**
**9:00 am - 12:00 noon**

**1:** Find all pairs of non-negative integers $x$ and $y$ such that $x - y = x^2 + xy + y^2$.

**2:** Evaluate $\displaystyle\int_0^\pi \ln(\sin x)\,dx$.

**3:** We are given $n \geq 4$ points in the plane such that the distance between any two of them is an integer. Prove that at least $\frac{1}{6}\binom{n}{2}$ of these distances are divisible by 3.

**4:** Evaluate $\displaystyle\sum_{k\in S} \left\lfloor \sqrt{\frac{n}{k}} \right\rfloor$, where $S = \left\{k \in \mathbf{N} \middle| \text{ for all } a \in \mathbf{N}, \text{ if } a^2 \text{ divides } k \text{ then } a = 1)\right\}$.

**5:** Let $L : \mathbf{R}^n \to \mathbf{R}^n$ be a linear transformation on $\mathbf{R}^n$, where $n$ is an integer greater than 1. Prove that there exists a two-dimensional subspace $V \subseteq \mathbf{R}^n$ such that $L(V) \subseteq V$.