# The Faculty of Mathematics at the University of Waterloo in association with The Centre for Education in Mathematics and Computing presents

## The Sixteenth Annual Small c Competition

for First and Second Year Students

Friday 23 September 2016

Time: 1 hour

Calculators are permitted.

**Instructions:** 

- 1. Do not open this booklet until you are told to do so.
- 10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
- 11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
- 100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
- 101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
- 110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
- 111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
- 1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.

There is no penalty for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 20.

- 1001. Diagrams are *not* drawn to scale. They are intended as aids only.
- 1010. Als u dit kunt lezen, spreekt u het Nederlands.
- 1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
- 1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
- 1001. The only website you may use during the contest is www.theonion.com.
- 1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
- 1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
- 10000. Hint: The answer to at least one question is **B**.
- 10001. If you finish early, please Brexit the room quietly.

### Part A

1. If u = 18 and v = 75, then  $\sqrt{\frac{2u}{3v}}$  equals

(A)  $\frac{2}{15}$ 

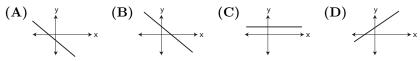
(B)  $\frac{2}{25}$ 

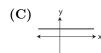
(C)  $\frac{2}{5}$ 

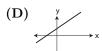
(D)  $\frac{4}{9}$ 

(E)  $\frac{4}{3}$ 

2. A reasonable sketch of the line  $y = -\frac{2}{3}x + 5$  is









3.  $2^24^48^8$  equals

(A)  $64^{14}$ 

(B) 64<sup>64</sup>

(C)  $2^{34}$ 

(D)  $2^{19}$ 

**(E)**  $2^{384}$ 

4. How many positive integers are less than  $20.17 \times 5$ ?

(A) 99

**(B)** 100

**(C)** 101

**(D)** 102

**(E)** 103

5. The top four scorers on the Small c contest for the past 15 years were given a cash prize. The chart below shows how each of these students chose to spend their money. What is the difference between the percentage of students who chose to spend their prize on video games and the percentage of those who chose to spend their prize money on textbooks? (Note that ## means five students so, for example, ##|| is a tally of seven students.)

Expense	Number of Students
Video Games	####
Fast Food	###
Textbooks	###
Music	## III

(A) 10

**(B)** 12

**(C)** 13

**(D)** 15

**(E)** 25

6. A tire has a radius of 23 cm. The distance between Kitchener and Toronto is about 110 km. Approximately how many complete rotations will the tire make in a round trip?

(A) 478 261

**(B)** 304 470

(C) 152 235

**(D)** 74 118

**(E)** 4873

7. Which of the following triples cannot be the lengths of the sides of a triangle?

(A) (2,5,6)

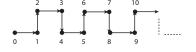
**(B)** (3, 4, 5)

**(C)** (5, 12, 13)

(D) (6,8,8)

**(E)** (2,3,6)

8. The "right, up, right, down" pattern shown to the right continues indefinitely. Which arrow configuration goes from point 2014 to point 2016? (This is not a rigorous definition of a pattern but after hours of deliberation, the Small c committee concluded that all reasonable people will see the same pattern.)



(A)

 $(B) \qquad (C) \qquad (D) \qquad (D)$ 

9. When Arcanine put 12 marbles in her bag, it was 1/2 full. When Blastoise put 9 marbles in his bag, it was 1/3 full. When Charizard put 7 marbles in her bag, it was 1/4 full. When Dragonite put 16 marbles in his bag, it was 2/3 full. When Electabuzz put 18 marbles in her bag, it was <sup>3</sup>/<sub>4</sub> full. Whose bag contains the most marbles when full?

(A) Arcanine

(B) Blastoise

(C) Charizard (D) Dragonite

(E) Electabuzz

10. If  $f(x) = x^2 - 9x + q$ , f(w) = -16, w = v, v = u, u = t, t = s, s = r, and r = q, then the value of f(-2) is

(A) 26

**(B)** 4

(C) -4

**(D)** 6

## Part B

- 11. In how many ways can 15 be written as the sum of four distinct elements from  $\{1,2,3,4,5,6,7,8,9\}$ ? (Order does not matter. That is, 1+3+4+7 and 3+1+7+4 do not count as different ways. Font size also does not matter.)
  - (A) 4
- **(B)** 5
- (C) 6
- **(D)** 7
- **(E)** 8
- 12. Given that  $\tan \theta = 1$  where  $0^{\circ} \le \theta \le 360^{\circ}$ , the sum of all possible values of  $\sin \theta$  is
  - (A) 1
- **(B)** 0
- (C)  $\sqrt{2}$
- (D)  $\frac{1}{\sqrt{2}}$
- 13. The two shorter sides of a right angle triangle measure  $\sqrt{2x^2+4x+2}$  and  $\sqrt{-x^2+2x+7}$ , where x is a positive real number. Determine the length of the hypotenuse in terms of x.
  - (A)  $x^2 + 6x + 9$

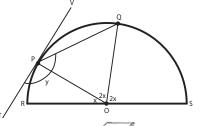
- (B)  $\sqrt{3x^2 + 2x 5}$  (E) x
- (C)  $\sqrt{2x^2+4x+2}+\sqrt{-x^2+2x+7}$

**(D)** x + 3

- 14. Point P and point Q lie on a semicircle with centre O and diameter RS. The measures of angles ROP, POQ and QOS are x, 2x and 2x respectively. Line segment TV is tangent to the semicircle at point P. What is the measure of obtuse angle TPQ? (Alternate wording: Look at the picture. What is y?)



- **(B)** 126°
- **(C)** 120°
- **(D)** 135°
- **(E)** 144°



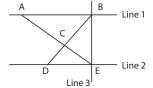
15. Cubes with volumes of 8, 64 and 729 are placed with faces touching as shown. What is the surface area of the resulting solid?

Before proceeding to the next question, pause to appreciate the high quality diagram.

- (A)  $606 \text{ m}^2$
- **(B)** 598 m<sup>2</sup>
- (C)  $582 \text{ m}^2$
- (**D**)  $566 \text{ m}^2$
- (E)  $558 \text{ m}^2$



- 16. The number of pairs (a, b) with  $a \in \mathbb{R}$  and  $b \in \mathbb{R}$  such that a + b ab 1 = 0 is
  - **(A)** 0
- **(B)** 1
- (C) 2
- **(D)** 100
- 17. Lines 1 and 2 are parallel. Line 3 is perpendicular to the other two. Line 4 does not exist. Points A, B, C, D, and E are as shown where AB = 32, DE = 8, and CD = 2. What is the distance between lines 1 and 2?



- (A) 6
- **(B)** 36
- (C) 13
- **(D)** 8
- **(E)** 10
- 18. Tonald Drump's three pet turtles are in a race. Turtle A comes in first, beating turtle B by 12 cm and turtle C by 15 cm. The race continues for second place, and turtle B beats turtle C by 5 cm. Assuming that each turtle races at a constant rate, how many centimetres long is the race?
  - (A) 32
- **(B)** 25
- (C) 75
- (D) 60
- **(E)** 30
- 19. Vulpix, Dragonair, Pikachu, Squirtle and Mewtwo are lined up at random. How many ways are there for Pikachu and Mewtwo to be next to each other in line?
  - (A) 16
- **(B)** 20
- (C) 36
- **(D)** 48
- **(E)** 120
- 20. In the village of Mathis Kewl, population 2016, the local lord numbers each villager from 1 to 2016 and arranges them in a circle in numerical order. He goes around the circle, removing villager #1, then villager #3, and so on, removing every other villager as he goes along. He continues to do this until only one villager remains whom he rewards by waiving their taxes for the year. What is the number of this lucky villager?
  - (A) 1024
- **(B)** 1968
- (C) 1984
- **(D)** 2000
- **(E)** 2016

### Part C

21. Following his break-up with Taylor, Tom decided to remodel his kitchen. He numbered the tiles on the floor as shown below and bought an irregular island in the shape given. He placed the island (without rotating it) so that the sum of the covered prime numbers was as small as possible. What is this sum?

(A) 17

**(B)** 20

**(C)** 41

**(D)** 14

**(E)** 47

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45



22. Let x and y be positive integers where x > 1. If both 44y + 865 and 53y + 1038 are multiples of x, what is x?

**(A)** 5

**(B)** 17

**(C)** 53

**(D)** 143

**(E)** 173

23. A square-based pyramid of height 3 cm and base length 2 cm is set on its square base. Water is poured through an infinitesimally small hole in the top of the pyramid to a height of 1 cm measured from the base, and then the hole is sealed. If the pyramid is turned upside-down, what will be the new height of the water?

Reminder: The volume of a square-based pyramid of height  $\Xi > 1$  and base length  $\Psi > 1$  is approximately

**(A)**  $\sqrt[3]{19}$ 

**(B)** 2

**(C)** 1

**(D)**  $\sqrt[3]{25}$ 

**(E)**  $\frac{76}{27}$ 

24. How many different 8-digit numbers can be formed using two 1s, two 2s, two 3s, and two 4s such that no two adjacent digits are the same?

(A) 720

**(B)** 792

**(C)** 840

**(D)** 864

**(E)** 1008

25. A box contains three white balls and two blue balls. HillIllary draws a ball from the box with replacement. If she draws a white ball, she wins a dollar and if she draws a blue ball, she wins two dollars. She can keep the money if and only if the total amount she has won is ever exactly \$100. If p is the approximate probability that HillIllary keeps the money, what is the  $32^{nd}$  digit after the decimal point in p?

Hint: This probability is less than the probability that there will be an NHL hockey team in Las Vegas in 2017.

**(A)** 4

**(B)** 1

**(C)** 6

**(D)** 7

**(E)** 2