

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Twenty-Third Annual Small c Competition

for First and Second Year Students

Friday, September 27, 2024

Time: 1 hour

Calculators are permitted.

Instructions:

1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
11. If you wish to cry, please do so quietly. Do not get any tears on the answer page.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must proudly display it.
101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A, B, C, D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty for an incorrect answer*.
Each unanswered question is worth 2, to a maximum of 20.
1000. Diagrams are *not necessarily drawn to scale*. They are intended as aids only.
1001. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
1011. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
1100. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
1101. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
1110. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
1111. Hint: The answer to at least one question is **B**.
10000. One bonus mark will be awarded to any contestant that gets Question 1 wrong and Question 25 right.
10001. Praising the Small c Competition on the subreddit [reddit.com/r/uwaterloo](https://www.reddit.com/r/uwaterloo) is permitted (and encouraged) as of 7 p.m. tonight.
10010. The only website you may use during the contest is [theonion.com](https://www.theonion.com).
10011. Due to an unfortunate virus, the 2020 edition of the Small c contest was deleted. If you recover this contest, please contact the CEMC immediately.
10100. Don't forget to turn off the Waterloo tap before you leave the Kitchener.

Part A

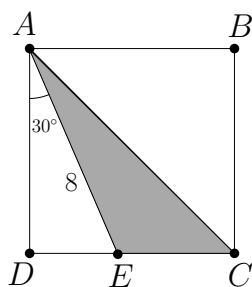
- Obligatory skill-testing question: what is the value of $12 + 24 \div 4 + 2$?
 (A) 20 (B) 6 (C) 11 (D) 16 (E) 21
- Bruno is a part-time student, and is therefore not eligible for the U-Pass. He buys a GRT fare card instead, and loads it with \$50 in stored value. Each trip on the transit system costs \$3, and Bruno cannot ride if his balance is lower than \$3. How many trips is Bruno able to take before he needs to reload his card?
 (A) 14 (B) 16 (C) 18 (D) 19 (E) 30
- Harder skill-testing question: when $2ab + (a - b)^2 - (a - b)(a + b)$ is simplified algebraically, what is the result?
 (A) $2b^2$ (B) 0 (C) $2ab$ (D) $2ab - b^2$ (E) $2ab + 2a - 2b - a^2 + b^2$
- If $x + 2y + 3z = 2$ and $3x + 2y + z = 6$, then what is the value of $x + y + z$?
 (A) -2 (B) -1 (C) 0 (D) 2 (E) 8
- In the table shown, the numbers 1 to 5 are written from left to right across the first row. Then, the numbers 5 to 9 are written right to left across the second row. The pattern continues, with consecutive integers written left to right and right to left in alternation, with the largest integer from the previous row becoming the smallest integer in the next row. In which column will the number 2024 appear?

A	B	C	D	E
1	2	3	4	5
9	8	7	6	5
9	10	11	12	13
17	16	15	14	13
\vdots	\vdots	\vdots	\vdots	\vdots

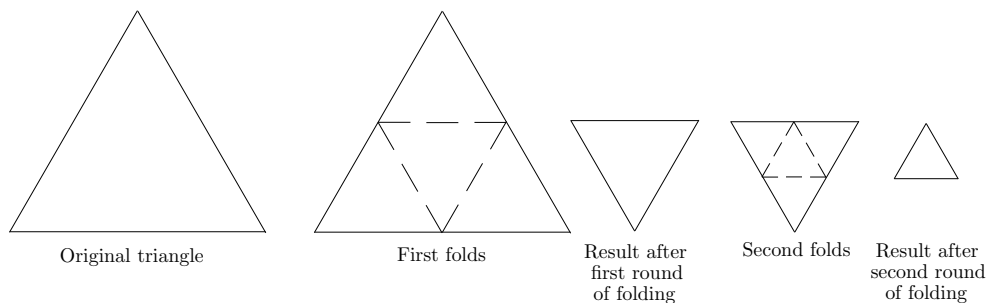
- (A) A (B) B (C) C (D) D (E) E
- Henrik made a circular mini-pizza, with radius 6 cm. Henrik cuts as many slices as he can with an outer arc of length 4 cm, and he stops when the final piece is too small to make another cut. If there are n slices in total ($n - 1$ the same and 1 smaller), what is the value of n ?
 (A) 2 (B) 4 (C) 5 (D) 10 (E) 18
 - Given that $3^{x+2} + 3^x = 90$, what is the value of $(5x)^x$?
 (A) 5 (B) 20 (C) 25 (D) 64 (E) 100
 - Charlotta is a Math student, and her friend Dunstan is a Geomatics student. Charlotta starts from Dunstan's position, walks 3 m north, 12 m east, and then another 3 m north. Dunstan correctly measures Charlotta's angle clockwise from north with a theodolite, while Charlotta uses a calculator and some trigonometry to get the correct answer. What answer do they both get, to the nearest tenth?
 (A) 22.6° (B) 24.6° (C) 63.4° (D) 65.4° (E) 67.4°
If you know what a theodolite is, give yourself a high-five.
 - Thirty windows in the Dana Porter library are labelled 1 to 30, with one light in each window. The lights in each window are initially turned off. Vivek turns the lights on in each window whose number is a multiple of 3. Mark then flips the switch in each window whose number is a multiple of 5, turning on lights that are off and turning off lights that are on. Finally, Mr. Goose comes and flips the switch in all the windows whose number is a multiple of 6. At the end of this process, how many windows have lights on?
 (A) 8 (B) 9 (C) 10 (D) 11 (E) 12
 - What is the value of $\lim_{x \rightarrow 0^+} \frac{x + \sqrt{x}}{x - \sqrt{x}}$?
 (A) -1 (B) 0 (C) 1 (D) ∞ (E) $-\infty$

Part B

11. A sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$. If the surface area to volume ratio is $1 \text{ cm}^2 : 3 \text{ cm}^3$, what is the radius of the sphere?
- (A) 1 cm (B) 3 cm (C) 4 cm (D) 6 cm (E) 9 cm
12. When the number 21^{2024} is written in standard form, what is the tens digit?
- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8
13. Five different integers in a list have a mean (average) of 40, and the difference between the largest and smallest integer is 12. Which of the following integers cannot be part of the list?
- (A) 32 (B) 36 (C) 41 (D) 43 (E) 49
14. After finishing with the lights in Dana Porter, Mr. Goose leaves the roof by flying 12 m north then 4 m east parallel with the ground, and then 3 m downwards toward the ground. Then, Mr. Goose realizes he forgot his coat, so he flies in a straight line back to his starting point. What is the total distance travelled by Mr. Goose?
- (A) 0 m (B) 12 m (C) 19 m (D) 32 m (E) 38 m
15. In square $ABCD$, point E lies along side CD , such that $\angle DAE = 30^\circ$ and $AE = 8$. What is the area of the shaded triangle ACE ?



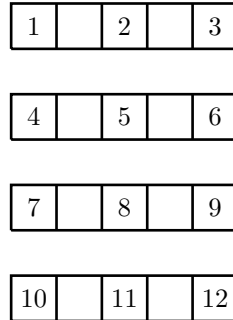
- (A) 24 (B) $24 - 8\sqrt{3}$ (C) $8\sqrt{3}$ (D) $24 + 8\sqrt{3}$ (E) $48\sqrt{3}$
16. Consider the equation $\log_2(x) = \log_x(2)$. What is the sum of all real values of x satisfying this equation?
- (A) 0 (B) 2 (C) $\frac{5}{2}$ (D) 4 (E) 5
17. A piece of paper in the shape of an equilateral triangle is folded from the corners to the centre of the opposite side three times to create a new, smaller equilateral triangle. The new triangle is then folded again to create a new, smaller equilateral triangle. This folding continues until the area of the triangle is less than 2% of the area of the original triangle, and then no more folding occurs. How many layers of paper thick is the resulting triangle?



- (A) 6 (B) 12 (C) 32 (D) 64 (E) 128

Distraction: what would the physical limit be to how many times we could fold this triangular paper?

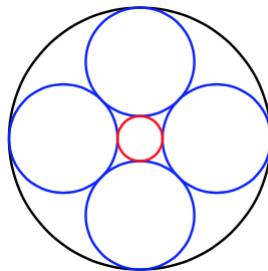
18. The Medium D Contest is invitation-only, with 12 invited students. These students write the contest in a room with 20 desks, in four rows of 5. In each row, only the middle desk and the desk on each end is occupied. Mr. Goose proctors the exam by taking paths starting from student 1 and flying from one occupied desk to the next, moving either one student to the right, or one student down in the diagram below, until arriving at student 12. How many different paths can Mr. Goose take from student 1 to student 12?



- (A) 3 (B) 6 (C) 10 (D) 12 (E) 35
19. A circle with centre O has five points marked on its circumference, $A, B, C, D,$ and E , in clockwise order. Points C and E are diametrically opposite one another (CE is a diameter). Furthermore, the line segments AE and OD are parallel. Given that $\angle DBE = 35^\circ$, what is the measure of $\angle ACE$?
- (A) 20° (B) 25° (C) 35° (D) 55° (E) 70°
20. Niraj has a poster printed for the Great Canadian Math Conference. One edge of the poster is 50 cm long. The poster is rolled starting from this edge, and placed into a cylindrical shipping tube with a length of 50 cm and a radius of 5 cm. The remaining air in the tube has the same volume as a cylinder of radius 4.5 cm and length 50 cm. If the poster is 0.2 cm thick, then how long is the remaining edge of the poster (to the nearest cm)?
- (A) 1 cm (B) 5 cm (C) 25 cm (D) 75 cm (E) 125 cm

Part C

21. The x -intercepts of the parabola $y = 3x^2 + bx + c$ have integer values exactly 4 units apart. If $-15 \leq c \leq 15$, how many distinct values are possible for c ?
- (A) 1 (B) 3 (C) 4 (D) 7 (E) 8
22. How many positive integers less than or equal to 2024 have the same number of positive factors as 2025?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
- Distraction: Did we need to specify "positive" factors in this question?*
23. Below is an arrangement of circles. The outside circle has radius 1. The four circles tangent to the outside circle have identical, smaller radii. If the circle in the middle is tangent to each of these four circles and has radius r , what is the value of $r^2 - 6r + 17$?



- (A) 0 (B) 13 (C) 15 (D) 16 (E) 20

24. In a randomly selected group of 41 students, 34 have studied in the DC Library, 27 have studied in the Dana Porter Library, and 24 have studied in the Math Comfy Lounge. A total of 13 of these students have studied in all three places. If n is the number of people who have studied in both libraries, but not the Comfy Lounge, what is the number of possible values of n ?

- (A) 4 (B) 8 (C) 9 (D) 17 (E) 18

25. A sequence is recursively defined by taking a real number c , and setting

$$\begin{cases} x_1 = c \\ x_{n+1} = x_n^2 + c, & \text{for } n \geq 1. \end{cases}$$

For some real numbers c , this sequence remains bounded, i.e. there is some constant R such that $|x_n| \leq R$ for all n . For example, when $c = 0$, $x_n = 0$ for all n , so the sequence is bounded.

If M is the greatest value of c for which the sequence is bounded, and m is the least value of c for which the sequence is bounded, what is $M - m$?

- (A) 1 (B) $\frac{5}{4}$ (C) 2 (D) $\frac{9}{4}$ (E) $\frac{5}{2}$