Calculus 1 Assignment 1

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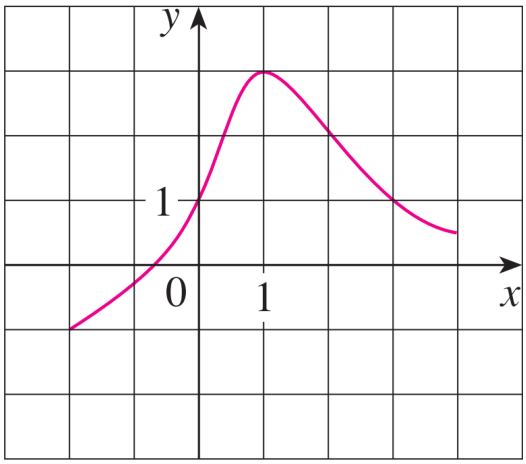
1 General Function Stuff

Problem 1.1: Suppose f is defined by $f(x) = x + x^2$, and g by $g(t) = t + t^2$. Is it true that $f = g$?

Problem 1.2: Suppose f is the function with the following graph:



a) What is $f(1)$?
b) What is $f(-1)$ (roughly)?
c) For what x is $f(x) = 1$?
d) For what x is $f(x) = 0$ (roughly)?
e) What is the domain of f?
f) What is the range of f?

Problem 1.3: Give an example of functions with the following properties. You can represent the functions however you like (including sketches), but it must be completely unambiguous which function you're talking about.

a) Domain $\mathbb{R}$, range $\{-1, 0, 1\}$.
b) Domain $(0, 1)$, range $[0, 1]$.
c*) Range $\mathbb{Q}$.

Problem 1.4: What is the domain of $f(x) = \frac{1}{(\sqrt{1-x}+1)(2x-3)}$?

2 Polynomials

Problem 2.1: A rock is dropped from a height. Its distance from the ground in meters is given by $h(t) = 5 - 5t^2$, where t is the time since it was dropped, in seconds.

a) What height was the rock dropped from?
b) When does the rock hit the ground?
c) Solve $h(t) = \frac{5}{2}$.
d) What height is the rock at 0.01 seconds before it hits the ground?

Recall (or learn for the first time) the following theorem:

Factor Theorem: Suppose that $p(x)$ is a polynomial, and that $p(r) = 0$ (i.e. r is a root of $p(x)$). Then $(x - r)$ is a factor of $p(x)$.

Example: Let $p(x) = x^2 - 1$. By inspection, $r = 1$ is a root, because we can see directly that $p(1) = 1^2 - 1 = 0$. The factor theorem then allows us to conclude for free that $(x-1)$ is a factor of $p(x)$. In fact, $x^2-1 = (x-1)(x+1)$; the factor theorem identified the first factor here. If we had also noticed that -1 was a root of $p(x)$, we would have gotten the complete factorization.

Problem 2.2: Fill in the missing entries in the following table.

$p(x)$	Factorization of $p(x)$	Roots of $p(x)$
$x^2 - 5x + 6$	a)	b)
$x^2 - x - 1$	c)	d)
$ax^2 + bx + c$	e)	f)

Problem 2.3: Expand $\left(x - \frac{-b+\sqrt{b^2-4c}}{2}\right)\left(x - \frac{-b-\sqrt{b^2-4c}}{2}\right)$.

3 Trigonometry

Problem 3.1:

a) Draw a square and one of its diagonals. Use this picture to explain why $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.
b) Find the values of $\sin\frac{\pi}{6}$ and $\cos\frac{\pi}{6}$ by drawing a picture of an equilateral triangle.

Problem 3.2: Draw a picture of the unit circle, and give the coordinates of the points with the following angles:

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{5\pi}{3}, \frac{3\pi}{2}, -\frac{\pi}{6}, -\frac{\pi}{4}, -\frac{\pi}{3}$

Problem 3.3: Use an identity to rewrite the following.

a) $\sin^2(x) + \cos^2(x)$
b) $\sin(2x)$
c) $\cos(2x)$