

Calculus 1 Assignment 10

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Due Friday, April 26th at 4 pm

1. Suppose $-2 \leq f(x) < 3$ for all x . What bounds can you put on $\int_5^{11} f(x) dx$?

2. Evaluate $\frac{d}{dx} \int_1^{x^2} f(t) dt$.

3. Evaluate the following integrals:

a) $\int x \sin(x^2) dx$

b) $\int \frac{1}{1+\log t} \frac{dt}{t}$

c) $\int_{-1}^{\pi} \log(x) e^{x^3-3} dt$

d) $\int_{-3}^0 e^{\frac{1}{x}} \frac{dx}{x^2}$

e) $\int_3^4 (x+1)^5 dx$

4. Write down Riemann sums which are reasonable approximations for the following integrals. These sums should be equal to actual numbers, as opposed to depending on any variables.

a) $\int_0^1 \log x dx$

b) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

c) $\int_{-1}^1 \cos(1000\pi\theta) d\theta$

5*. In class I gave the definition

$$\int_a^b f(x) dx := \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{b-a}{n} f\left(a + k \frac{b-a}{n}\right).$$

This is in fact not an appropriate definition of an integral. What is true is that if the integral on the left exists (according to the definition that mathematicians actually use), then it is equal to the limit on the right. However, it can be the case that, for extraordinarily messy functions, the limit on the right exists, but the integral on the left does not exist. In this question we'll explore one such example.

Let f be our loyal companion

$$f : \mathbb{R} \rightarrow \{0, 1\}, \quad f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q} \end{cases}.$$

a) Using the definition given in class, evaluate

$$\int_0^1 f(x) dx.$$

b) Let α be an arbitrary irrational number in the interval $(0, 1)$. Evaluate

$$\int_0^{\alpha} f(x) dx + \int_{\alpha}^1 f(x) dx.$$

c) Explain why the definition of the integral that I gave in class is not the one that is used by mathematicians.
d) How would you choose to define integrals?