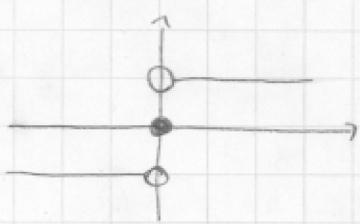
Assignment 1 Solutions.

1.1. I didn't specify the domains of these functions, but f and g are equal if they have the same domain. They are equal because for every input the output of the two functions is the same. The choice to use x vs t is irrelevant; all the notation is saying is that the output is the input plus the square of the input.

1.2: a) 3
b) $-1/3$
c) 0 and 3
d) $-2/3$
e) $[-2,4]$
f) $[-1,3]$

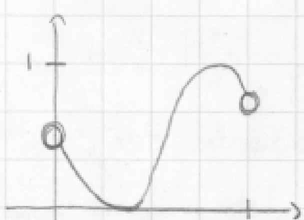
1.3. There are of course many possible answers. Here are some.

a)



$$f(x) = \begin{cases} -1, & x<0 \\ 0, & x=0 \\ 1, & x>0 \end{cases}$$

b)



c) $f: \mathbb{Q} \to \mathbb{Q}$, $f(x) = x$ (i.e. just restrict the domain to $\mathbb{Q}$)

1.4. Here, of course, I mean the largest possible domain contained in $\mathbb{R}$.

Considerations: I can't divide by 0, and I can't put a negative number under the square root. So my conditions are:

(1) $1-x \geq 0$ (because $\sqrt{1-x}$ appears)

(2) $2x-3 \neq 0$ (because then I'd be dividing by 0)

(3) $\sqrt{1-x}+1 \neq 0$ (because then I'd be dividing by 0)

Condition (1) means I need $x \leq 1$.

Condition (2) means I need $x \neq \frac{3}{2}$

Condition (3) is always satisfied, because $\sqrt{u} \geq 0$ for every u.

So overall the biggest domain possible is $(-\infty, 1]$.

2.1 a) 5m. The rock is dropped at $t=0$, and $h(0)=5$.

b) $t=1$s. The rock hits the ground when $h(t)=0$, and by inspection $h(1)=0$.

c) $5-5t^2 = \frac{5}{2} \iff 1-t^2 = \frac{1}{2} \iff t^2 = \frac{1}{2} \iff t = \frac{1}{\sqrt{2}}$. The other root, $t = -\frac{1}{\sqrt{2}}$, doesn't really make sense in this context.

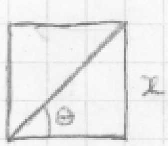
d) 0.01s before the rock hits the ground is $t = 0.99$. Evaluating, $h(0.99) = 0.0995$

2.2 a) $(x-2)(x-3)$ b) 2, 3

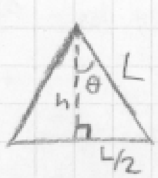
c) $\left(x - \frac{1+\sqrt{5}}{2}\right)\left(x - \frac{1-\sqrt{5}}{2}\right)$ d) $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

e) $\left(x - \frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x - \frac{-b-\sqrt{b^2-4ac}}{2a}\right)$ f) $\frac{-b+\sqrt{b^2-4ac}}{2a}, \frac{-b-\sqrt{b^2-4ac}}{2a}$

2.3. You could multiply this out, but you can also just notice that $\frac{-b \pm \sqrt{b^2-4c}}{2}$ are the roots of x^2+bx+c, so the factor theorem tells us that multiplying out the expression in the question will give us something $\cdot (x^2+bx+c)$, and then matching degrees and leading coefficients tells us that we get x^2+bx+c exactly.

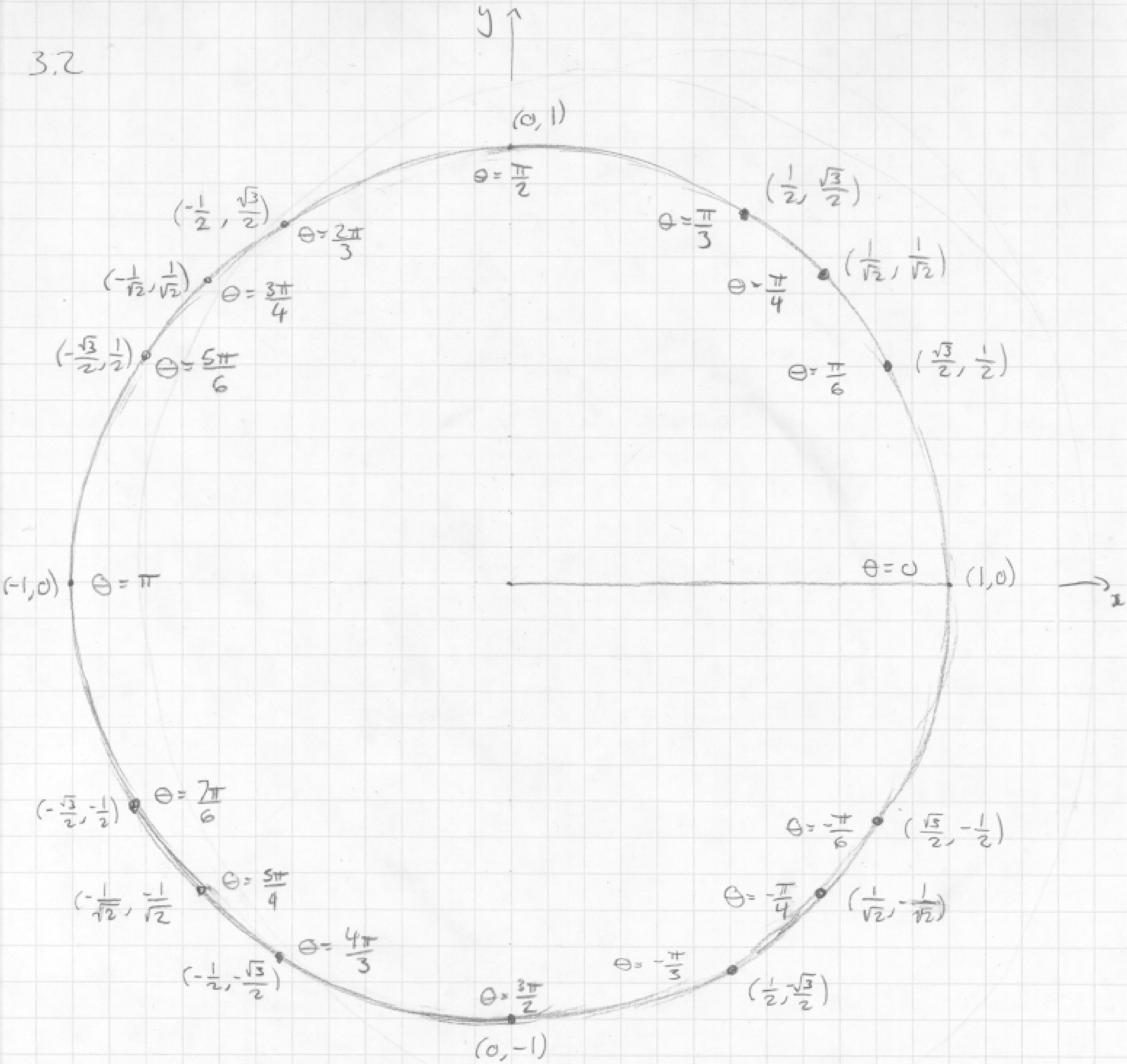
3.1 a) 

If the length of a side of the square is x, then the length of the diagonal is $\sqrt{x^2+x^2} = \sqrt{2}x$. The angle θ is $\frac{\pi}{4}$, since $2\theta = \frac{\pi}{2}$. $\sin\theta$ is the ratio of the length of the side opposite θ to the length of the hypotenuse, so $\sin\frac{\pi}{4} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$. Similarly for $\cos\frac{\pi}{4}$.

b) 

The angle θ is $\frac{\pi}{6}$, because $2\theta = \frac{\pi}{3}$. The height h satisfies $h^2 + (L/2)^2 = L^2$, so $h = \frac{\sqrt{3}}{2}L$. Thus $\sin\frac{\pi}{6} = \frac{L/2}{L} = \frac{1}{2}$, and $\cos\frac{\pi}{6} = \frac{\frac{\sqrt{3}}{2}L}{L} = \frac{\sqrt{3}}{2}$.

3.2



3.3. a) $\cos^2 x + \sin^2 x = 1$

b) $\sin(2x) = 2\cos x \sin x$

c) $\cos(2x) = \cos^2 x - \sin^2 x$