Calculus 1 Assignment 3

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1 Rock Throwing

Problem 1.1: Alice drops a rock from a height of 5 meters in an alternate universe. In this alternate universe, the height of the rock above the ground is given by $h(t) = 5 - 5t^3$. How fast is the rock falling when it hits the ground? Only use arguments similar to those discussed in class and in the first two chapters of Stewart.

2 Limits

Problem 2.1: What are the following limits?

a) $\lim_{x\to 0} \frac{\sin(\pi x)}{x}$

b) $\lim_{x\to 0} \frac{\log(ax+1)}{x}$, where a is an arbitrary real number. (Hint: experiment with some of fixed values of a.)

c) $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$

d) $\lim_{x\to 0} \frac{x}{|x|}$

e) $\lim_{x\to 0} f(x)$, where $f(x) = 0$ if $x \notin \mathbb{Q}$, and $f(x) = 1$ if $x \in \mathbb{Q}$.

f*) $\lim_{x\to \frac{1}{2}} f(x)$, where $f(x) = 0$ if $x \notin \mathbb{Q}$, and $f(x) = \frac{1}{q}$ if $x = \frac{p}{q}$ is a fraction in lowest terms and $q > 0$.

2.2: Give an example of a function f and a point a such that $f(a)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a^-} f(x)$ all exist and are all unequal.

2.3: Suppose f and g are "nice" functions. Show that

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} = f(x) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} + g(x) \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

What assumptions did you have to make about f and g (i.e. what does "nice" mean here?)

(Hint: Add and subtract $\lim_{h\to 0} \frac{f(x+h)g(x)}{h}$ to the left hand side.)