Calculus 1 Assignment 4

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Due Wednesday, February 20th 5 pm

1.
a) Determine whether or not the function $x^4 + 3x + 1$ has a real root.
b) Show that there are two antipodal points on the equator that have the same temperature. What assumption did you have to make about how temperature behaves in the physical world?

2. Evaluate the following limits:

a)
$$\lim_{x \to -1} \frac{x^2 - 3x - 4}{x^2 + 10x + 9}$$

b)
$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - \frac{9}{2}x + 5}$$

c)
$$\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^3 - 1}$$

d)
$$\lim_{x \to -\infty} \frac{4x^3 + 27x^2 + 1096x - \sin x}{3x^3 + \frac{16}{25}x + x^{-2019}}$$

e)
$$\lim_{x \to -\infty} \frac{4x^3 + 27x^2 + 1096x - \sin x}{3x^4 + \frac{16}{25}x + x^{-2019}}$$

f)
$$\lim_{x \to -\infty} \frac{4x^5 + 27x^2 + 1096x - \sin x}{3x^4 + \frac{16}{25}x + x^{-2019}}$$

g)
$$\lim_{x \to \infty} \frac{e^{-x}}{\arctan x \cdot \log x}$$

h)
$$\lim_{u \to 0} \frac{\cos(u^2 - 1)}{(\sqrt{1-u}+1)(2u-3)}$$

i)
$$\lim_{x \to \infty} \frac{\cos(x^{-2} - 1)}{\left(\sqrt{1-\frac{1}{x}}+1\right)\left(\frac{2}{x}-3\right)}$$

j)
$$\lim_{x \to \infty} \cos x$$

3.
a) Give an example of a function f which is continuous at a point a but not differentiable at a.
b) Why is it that if f is discontinuous at a then it cannot be differentiable at a?

4. What is the equation of the line tangent to the function $f(x) = 5 - 5x^2$ at the point $(1,0)$?