3. Using the definition of the derivative:

$$\begin{aligned} f'(0) &= \lim_{\delta\to 0} \frac{f(0+\delta)-f(0)}{\delta} \\ &= \lim_{\delta\to 0} \frac{\delta^2 \sin\left(\frac{1}{\delta}\right) - 0}{\delta} \\ &= \lim_{\delta\to 0} \delta \sin\left(\frac{1}{\delta}\right). \end{aligned}$$

This limit is the limit from assignment 3, problem 2.1 c). It equals 0.

$$\begin{aligned} g'(0) &= \lim_{\delta\to 0} \frac{g(0+\delta)-g(0)}{\delta} \\ &= \lim_{\delta\to 0} \frac{\delta \sin\left(\frac{1}{\delta}\right) - 0}{\delta} \\ &= \lim_{\delta\to 0} \sin\left(\frac{1}{\delta}\right). \end{aligned}$$

This limit was given in class when introducing limits. It doesn't exist.

$$\begin{aligned} h'(0) &= \lim_{\delta\to 0} \frac{h(0+\delta)-h(0)}{\delta} \\ &= \lim_{\delta\to 0} \frac{\begin{cases} \delta^2 & \text{if } \delta \text{ is rational} \\ 0 & \text{if } \delta \text{ is irrational} \end{cases} - 0}{\delta} \\ &= \lim_{\delta\to 0} \begin{cases} \delta & \text{if } \delta \text{ is rational} \\ 0 & \text{if } \delta \text{ is irrational} \end{cases}. \end{aligned}$$

To go from the second line to the third line, observe that for every δ the two expressions give the same result. Evaluating this limit can be done similarly to problem 3.2.3 on the midterm. Using the squeeze theorem, with an upper function of $|\delta|$ and a lower function of $-|\delta|$ shows that this limit is 0.

4. There are many ways to approach this problem depending on how you like to think about the derivative. I think of the derivative primarily as a rate of change, so I would try to estimate the rate of change of T near $x = 1$. We're given that $T(0.99) = -0.0101$ and $T(1.01) = 0.0099$, so T increases by 0.02 as x goes from 0.99 to 1.01. The gap between 0.99 and 1.01 is 0.02, so I would estimate that $T'(1) = \frac{0.02}{0.02} = 1$.

One consideration is that this function might wiggle a bit, so I'm inclined to check that the rate of change of T is 1 by looking at the other values and seeing if they cooperate with that hypothesis. Going from 0.98 to 0.99, the value of T increases by about 0.01, as it should if the derivative is about 1, and going from 1.01 to 1.02 similarly causes T to increase by about 0.01. As you get farther from $x = 1$ the rate of change starts being a bit off from 1, but not in an abnormal way.

There are several other ways to do think about this problem. For example, if you are inclined to interpret $T'(1)$ as the slope of the tangent line, then you could pick two points on the curve with x coordinates near 1 and draw a line through them. The line you get will be pretty close to the tangent line, so you can approximate $T'(1)$ with the slope of the line you constructed. If you do this using the two points with x coordinate nearest to 1 then you end up doing the same calculation I did above.

The values of T given are just the values of log truncated to 4 digits, so this can also give you a pretty reasonable idea of what kind of variation is normal in a function.