Calculus 1 Assignment 7

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Due Wednesday, March 27th at 5 pm

1. Draw a picture of a function $f : [0,1] \to \mathbb{R}$ that is continuous on its domain and differentiable on $(0,1)$, and then use that picture to illustrate that the Mean Value Theorem (MVT) holds for this function.

2. Suppose f is a function from $[0,1]$ to $\mathbb{R}$. Is it possible for f to be differentiable at 0? (Give an example of a function $f : [0,1] \to \mathbb{R}$ which is differentiable at 0, or prove that no such function can exist.)

3. Give examples of functions $f : [0,1] \to \mathbb{R}$ which are continuous on
a) $(0,1]$
b) $[0, \frac{1}{2})$ and $(\frac{1}{2}, 1]$
for which the conclusion of MVT does not hold.

4. Give an example of a function $f : [0,1] \to \mathbb{R}$ which is continuous on its domain and differentiable on $(0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$ for which the conclusion of MVT does not hold.

5. Find all local minima, local maxima, global minima, and global maxima of the following functions:
a) $\sin(x)$, $x \in \mathbb{R}$
b) $\sin(x)$, $x \in (\frac{\pi}{6}, \frac{5\pi}{6})$
c) $|\sin(x)|$, $x \in [-4, 4]$
d) $3x^4 + 4x^3$, $x \in [-2, 5)$
e) $\frac{1}{x}$, $x \in \mathbb{R} \backslash \{0\}$

6. Find two numbers whose difference is 100 and whose product is a minimum.

7. Find two positive numbers whose product is 100 and whose sum is a minimum.