

# Calculus 1 Assignment 9

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Due Friday, April 19th at 4 pm

**1.**

- a) Write down a sum which estimates the area under the curve  $y = x^2$  between  $x = 0$  and  $x = 1$ .
- b) Write down a limit which is exactly equal to the area under the curve  $y = x^2$  between  $x = 0$  and  $x = 1$ , and then evaluate that limit. You will probably need to google “sum of consecutive squares” at some point.

Fun fact: Archimedes figured this out! He’s most well known for his “eureka” moment, when he figured out how to determine what metal the crown of the king was made out of, but the accomplishment he was most proud of was his ability to determine areas of shapes like these. He asked for a depiction of his formula for the volume of a sphere, which he did in a similar way, to be engraved on his tombstone. He was right to be so proud, as he essentially invented integral calculus almost two millennia before anyone else.

**2.**

- a) Estimate the length of the curve  $y = x^2$  between  $x = 0$  and  $x = 1$ . For example, a first approximation might be that the length of the curve is just the distance between  $(0, 0)$  and  $(1, 1)$ , which is  $\sqrt{2}$ . If you draw lines between  $(0, 0)$  and  $(\frac{1}{2}, \frac{1}{4})$ , and then between  $(\frac{1}{2}, \frac{1}{4})$  and  $(1, 1)$ , and take the length of those two lines, you get a better approximation of the length of the curve.
- b) Write down a limit whose value is the exact length of the curve  $y = x^2$  between  $x = 0$  and  $x = 1$ . This limit will be hard to evaluate (and you don’t have to evaluate it), but write down exactly what it is.

**3.** In class we discussed the road trip of Alice, Bob, and Charles. The speed of their very strange car was given by

$$v(t) = 20 \frac{\text{mi}}{\text{h}^2} t$$

between  $t = 0$  and  $t = 3\text{h}$ . They estimated the distance that they travelled during this trip by

1. splitting the trip into small, “parts”,
2. making the approximation that their speed throughout each part of the trip was equal to their speed at the end of that part of the trip,
3. estimating the distance travelled during each part of the trip using Distance = Speed  $\times$  Travel time, and
4. summing up each of the distances found in the previous step.

In step 2, they made a choice of using their speed at the end of that part of the trip to approximate their speed throughout the part. Instead, they could have used their speed at the beginning of that part of the trip, for example. Does this choice matter? There are several ways to justify your answer, including redoing the calculation with the other choice or drawing a picture.