

Calculus 1 Practice Problems

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1.* Wolfram Alpha can tell you the actual number, and you should be within about 100.

2. $x = 2$.

3. $\sum_{k=1}^{10000} \frac{12}{10000} e^{-(-10+12\frac{k}{10000})^2}$ is one such sum.

4.*

a) $\frac{6000}{7}$.

b) $\frac{5000}{6}$.

c) $\frac{3000}{4}$.

d) $\frac{\int_0^1 xf(x) dx}{\int_0^1 f(x) dx}$.

5.* $\frac{2}{\pi}$.

6. $1 - \frac{0.3}{n}$.

7.* Use Wolfram Alpha to check your answers.

8.* Wolfram Alpha.

9. Wolfram Alpha.

10. Wolfram Alpha. (This was done in class)

11. Wolfram Alpha.

12. Wolfram Alpha.

13.

a) The point $(x, f(x))$ is a global maximum if $f(x) \geq f(y)$ for every y in the domain of f . The point $(x, f(x))$ is a local maximum if there exists an open interval I on which it is a global maximum. Similarly for minima.

b) One such function is

$$\begin{cases} \frac{1}{2} - x & 0 \leq x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ \frac{3}{2} - x & \frac{1}{2} < x \leq 1 \end{cases}$$

c) One such function is

$$\begin{cases} x + 2 & x < -1 \\ -x & -1 \leq x \leq 1 \\ x - 2 & x > 1 \end{cases}$$

d) One such function is $\sin x + x$.

14. About -0.91 .

15. $y = -\frac{1}{3}x + 8$.

17. See the Wikipedia article on related rates.

18. $\frac{16(\sqrt{2} - \sqrt{3}\pi \log 3)}{\pi(8\sqrt{2} - \sqrt{3}\pi^2 \log 3)}$.

(Note: I just plugged this one and the next two into Wolfram Alpha and didn't think too hard about it. Please let me know if you think this answer is wrong.)

19. $\frac{6(\sqrt{2} - \pi)}{2 + \pi}$.

20. $-\frac{18 \log 2}{1 + 24 \log 2}$.

21.

a) $x = -3, -1, 3$, and every number in the interval $(1, 2)$.

b) $x = -4.5, -1.75, 4.5$, and every number in the interval $(1, 2)$.

22. Wolfram Alpha.

23. $\frac{e^t}{1 + e^{4t}}$.

24. False.

25. False.

26. Wikipedia.

27. Use the fact that $\int_M^\infty \frac{dx}{x^2} = \frac{1}{M}$.

28. Both false.

29. True (for example $f(x) = |x|^{-\frac{1}{2}}$ between -1 and 1).

30-32. IVT.

33.

a) MVT.

b) Suppose otherwise and then use part a).

c) If $f < g$ then $\int f < \int g$. Apply this to $f' > 0$ and use FTC.

34.

a) $a + b = 4$.

b) $a = 4, b = 0$.

35. All Wolfram Alpha, except for

h) 1

i) $\tan \sqrt{2}$

36. Wolfram Alpha

37. Wolfram Alpha