Math 218 — Assignment 4

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Due 2024/10/04

Give your code for all problems on this assignment.

1. Suppose $z(x)$ satisfies the differential equation $z' = \cos(100x^2 + z^3)$ and that $z(0) = 1$.

a) Write a computer program to numerically estimate the value of $z(10)$. What value do you get? How long did your code take to run?

b) Describe the behaviour of $z(x)$. Explain it qualitatively.

c) Try to estimate the value $z(x_m)$ for x_m as large as possible. How large can you get x_m to be? Give one reason why you think your value of $z(x_m)$ is reasonably accurate, yet you wouldn't be able to provide an accurate value of, say, $z(10x_m)$.

d*) Consider the use of a dynamic step size. What are advantages and disadvantages here? What if instead $z' = \cos(100x^{2.1} + z^3)$?

2. Consider the forced damped simple harmonic oscillator $\ddot{y} + 0.1\dot{y} + 2y = \cos\omega_r t$, with initial conditions such that the homogeneous part of the solution is 0, and driven at its resonance frequency.

a) What is the resonance frequency ω_r of this system, and what is the corresponding amplitude?

b) How many oscillations does it take for y to first reach 99% of its maximum amplitude?

3. Consider the initial value problem $y' = -ty + 0.1y^3$ and $y(0) = \alpha$, where α is a given number. There is a critical value $\alpha = \alpha_0$ that separates converging solutions from diverging ones. Estimate α_0 numerically.

4. Investigate and describe the behaviour of the system $y' = \frac{3t^2}{3y^2-4}$.