Math 218 — Assignment 7

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Due 2024/11/11

1. Prove the following.

1.1) $\mathscr{L}\{af(t) + bg(t)\} = a\mathscr{L}\{f(t)\} + b\mathscr{L}\{g(t)\}$.

1.2) $\mathscr{L}\{e^{at}y(t)\}(s) = \mathscr{L}\{y\}(s-a)$.

1.3) $\mathscr{L}\{y(at)\} = \frac{1}{a}\mathscr{L}\{y\}\left(\frac{s}{a}\right)$ when $a > 0$. What goes wrong when $a \leq 0$?

1.4) $\mathscr{L}\{t^n y(t)\} = (-1)^n \frac{d^n}{ds^n}\mathscr{L}\{y\}$.

1.5) $\mathscr{L}\{\mathbb{1}_{\{t>c\}}y(t-c)\} = e^{-cs}\mathscr{L}\{y\}$, where $\mathbb{1}_{\{t>c\}} := \begin{cases} 1 & t > c \\ 0 & \text{otherwise.} \end{cases}$

2. Solve the following differential equations using Laplace transforms.

2.1) $y'' + y = \sin 2t$ with $y(0) = 2$ and $y'(0) = 1$.

2.2) $4y'' + 4y' + 5y = \begin{cases} \sin(t) & \pi \leq t < 2\pi \\ 0 & \text{otherwise} \end{cases}$ with $y(0) = y'(0) = 0$.

2.3) $y'''' - y = \delta(t-1)\log(t+1)$ with $y(0) = y'(0) = y''(0) = y'''(0) = 0$. Here δ denotes the Dirac delta function.

2.4) $y'' + y = t^3$ with $y(0) = y'(0) = 0$. Use the convolution theorem.

3. When y is infinitely differentiable everywhere it is the case that

$$\mathscr{L}\left\{\frac{d^n}{dt^n}y(t)\right\} = s^n\mathscr{L}\{y\} - \sum_{k=1}^{n} s^{n-k}\frac{d^{k-1}y}{dt^{k-1}}\bigg|_{t=0}.$$

What about when y is only piecewise infinitely differentiable?