

Math 288X — Assignment 2

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Due 2023/09/22 3pm in class or by email

1. Let X be a positive real number. Define

$$\varphi(s) := \frac{\Gamma(\frac{s}{2} - 1) X^s}{\zeta(\frac{s}{2}) s}.$$

Assume that ζ' and ζ'' are nonzero at the zeros of ζ .

1.1. List the locations and orders of all the poles of φ .

1.2. For each pole s_0 of φ with $\Re(s_0) \geq -6$, express $\operatorname{Res}_{s=s_0} \varphi(s)$ in terms of Γ , ζ' , and ζ'' .

1.3.* Express the values of the remaining residues in terms of ζ' , ζ'' , Bernoulli numbers, and harmonic numbers.

1.4. Devise, perform, and describe any sort of sanity check for any of your answers.

* Feel free to skip or leave incomplete any questions, particularly 1.3.