Math 288X — Assignment 3

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Due 2023/09/29 3pm in class or by email

1. Let χ be a nontrivial Dirichlet character, w a complex number, and n a positive integer. Define

$$\varphi(n, w, \chi) := n^w \prod_{\substack{p \mid n \\ p \text{ prime}}} 1 - \frac{\chi(p)}{p^w}.$$

Assume the Riemann hypothesis for the Riemann zeta function and Dirichlet L-functions.

1.1. Suppose $\Re(w) > 0$. Give an asymptotic for the partial sum $\sum_{n<X} \varphi(n, w, \chi)$ with a power-saving error term, i.e. prove a statement of the form

$$\sum_{n<X} \varphi(n, w, \chi) = cX^\alpha + \mathcal{O}(X^\beta)$$

for some α, β, c with $\Re(\alpha) > \Re(\beta)$.

1.2.* Repeat problem 1.1 with the assumption that $-\frac{1}{2} < \Re(w) < 0$. For a lower bound on L-functions in the critical strip, you can use Iwaniec–Kowalski theorem 5.19.