

# Math 288X — Assignment 4

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Due 2023/10/13 3pm in class or by email

1. Let  $p$  and  $q$  be distinct primes. Let  $\psi_0$  denote the trivial character mod  $pq$ , and  $\mathbf{1}$  the trivial character mod 1. Let  $E_{\chi_1, \chi_2}(z, s)$  denote the weight 0 “Eisenstein series attached to characters” from Young 2019 - *Explicit calculations with Eisenstein series*.

1.1. What are the cusps of  $\Gamma_0(pq)$ ? Which one is equivalent to  $i\infty$ ? (You can find this in Prop. 3.3.8 of Goldfeld–Hundley, in Prop. 2.6 of Iwaniec - *Topics in Classical Automorphic Forms*, and in Young 2019 §5.2.)

1.2. Write  $E_{i\infty}(z, s, \psi_0)$  in terms of  $E_{\mathbf{1}, \mathbf{1}}$ . (Theorem 6.1 of Young 2019)

1.3. Give the Fourier expansion of  $E_{\mathbf{1}, \mathbf{1}}(z, s)$ . (Prop. 4.1 of Young 2019)

1.4. Give the Fourier expansion of  $E_{i\infty}(z, s, \psi_0)$ .

1.5. Skim §2.1 of Hoffstein–Lee 2014 - *Shifted Multiple Dirichlet Series*, <https://arxiv.org/abs/1412.5917>.