Math 288X — Assignment 7

Alex Cowan

Due 2023/11/10 3pm in class or by email

Let

$$\mu(z) = \sum_{n\neq 0} 2\rho(n) y^{\frac{1}{2}} K_{ir}(2\pi|n|y) e(nx)$$

be a Maass form of level 1 and Hecke eigenform, let $E(z,w)$ be the level 1 Eisenstein series, and let $P_\ell(z,s)$ be the level 1 Poincaré series, where ℓ is a positive integer.

1. Write $\langle E(z,w)\overline{\mu(z)}, P_\ell(z,\bar{s})\rangle$ in terms of an integral over $\Gamma_\infty\backslash\mathcal{H}$.

The integral from the previous problem doesn't appear in Gradshteyn and Ryzhik. The only table I've found it in is Brychkov–Marichev–Savischenko 3.14.19.1, and it's not in a very convenient form. Sizeable parts of the example-setting papers Takhtadzhyan–Vinogradov *The zeta function of the additive divisor problem and spectral expansion of the automorphic Laplacian* and Jutila *The additive divisor problem and its analogs for Fourier coefficients of cusp forms I* are dedicated to technical analysis of this integral.

2.* Can you think of another construction, not involving P_ℓ, that'd yield a shifted convolution of the Fourier coefficients of E and μ? Hint (rot13): Guvf qbrf abg vaibyir na vaare cebqhpg.

3. Evaluate $\langle E(z,w)\overline{\mu(z)}, \mu'(z)\rangle$ and $\langle E(z,w)\overline{\mu(z)}, E(z,\bar{\lambda})\rangle$, where μ' is another level 1 Maass form and Hecke eigenform, and λ is a complex number with real part $\frac{1}{2}$. Must you impose any assumptions on w?

4. Why might the inner products from the previous problem be useful for studying $\langle E(z,w)\overline{\mu(z)}, P_\ell(z,\bar{s})\rangle$ or the quantity from the second problem?