CO781 HW set#2.

(1) Show that the bitwise H and CNOT for the 7-bit Steane code satisfy properties Ga FT 1 and 2. [4 marks]

(2a) An erasure error takes a qubit state to an erasure symbol orthogonal to the qubit space. Show that a distance d (in terms of the Pauli error basis) quantum code can correct for d-1 erasure errors. [3 marks]

(2b) Consider the stabilizer code with stabilizer generated by XXXX and ZZZZ. Give a short argument that the distance is 2, and it encodes 2 qubits. [2 marks] Write down the encoded X and Z for the two logical qubits (the answer is not unique, take any that is correct). [2 marks] If Bob has 4 daughters, and encodes 2 qubits in this code, and gives the ith qubit to the ith daughter, what data can any 3 recover? [1 mark] What data can be covered by any 2 of them? [2 marks]

(3) Find the encoded X and Z for the 5-bit code. [1 mark] What happens if the bitwise CNOT or the bitwise H is applied? [2 marks] How to perform an encoded CNOT and encoded H given the ability to measure product of unencoded Pauli operators? Justify the procedures. [6 marks]

23 marks so far. This HW set is supposed to have 25 marks. You can get up to 5 marks for one of the following:

(4) Read 10.4.2 and do ex 10.27.

(4') Consider the 9-bit code with stabilizer:

S1=XXXXXXIII
S2=IIIXXXXXX
S3=ZZIIIIIII
S4=IZZIIIIII
S5=IIIZZIIII
S6=IIIIZZIII
S7=IIIIIIZZI
S8=IIIIIIIZZ

with N(S)/S generated by XXXXXXXXX, ZZZZZZZZZ.
It has distance 3 and encode 1 qubit. Note that the Z error on the 1st and 2nd qubits act identically on the codespace and we don't need to distinguish between them.

If we replace S3 by S3*S5*S7, and S4 by S4*S6*S8, and remove S5,S7,S6,S8, the new stabilizer has 4 generators. So, the subspace stabilized has 32 dimensions. what generates the new N(S)/S? (The answer is not unique, but certain generating sets are more obvious than the others. For example, you should certainly keep XXXXXXXXX and ZZZZZZZZZ.)

The 8 new generators in N(S)/S represent encoded X and Z on the 4 extra qubits. Are Z acting on the 1st and 2nd qubit still degenerate?

Actually these 4 extra encoded qubits are not protected from 1-qubit errors. The original encoded qubit is. If we apply bitwise H, the stabilizer is not preserved. But researchers have used it as encoded H nonetheless, how can that be?