

If we have two codeblocks each in a stabilizer code (with respective stabilizers  $S$ ,  $T$ , generators  $S_i$ ,  $T_j$ 's, and acting on two ambient spaces  $A_1$  and  $A_2$ ). Let the first codeblock has  $k_1$  qubits encoded in  $n_1$  qubits, the second codeblock has  $k_2$  qubits encoded in  $n_2$  qubits. So, there are  $n_1 - k_1$   $S_i$ 's and  $n_2 - k_2$   $T_j$ 's.

What is the stabilizer for the two codeblocks together?

If  $s \in S$  and  $t \in T$ ,  $s \otimes t$  stabilizes the combined codeblock. Take  $s=I$  and  $T_j \in T$  (where  $I$  is the big identity on  $A_1$ , and is just the tensor product of  $n_1$  qubits) and  $s=S_i$  and  $t=I$ . So,  $S_i \otimes I$  and  $I \otimes T_j$  stabilize any state in the two codeblocks, and there are  $[n_1 - k_1 + n_2 - k_2]$  generators.

Furthermore, together, there are  $k_1 + k_2$  qubits encoded in  $n_1 + n_2$  qubits. So, the above already gives the maximum number of generators for the joint stabilizer.

For the 2 blocks of 7 bit code:

```
IIIXXXX IIIIIII
IXXIIXX IIIIIII
XIXIXIX IIIIIII
```

```
IIIZZZZ IIIIIII
IZZIIZZ IIIIIII
ZIZIZIZ IIIIIII
```

```
IIIIIII IIIXXXX
IIIIIII IXXIIXX
IIIIIII XIXIXIX
```

```
IIIIIII IIIZZZZ
IIIIIII IZZIIZZ
IIIIIII ZIZIZIZ
```

Call the qubits 1-14, and with an abuse of language, called the above the stabilizers  $S_1-6$ ,  $T_1-6$  instead (so, they become 14-qubit Pauli operations, not 7-qubit Pauli operations).

Perform CNOT from qubit 1- $\rightarrow$ 8, 2- $\rightarrow$ 9, ..., 7- $\rightarrow$ 14.  
(So, we need to look at qubits 1,8, qubits 2,9, etc, and combine the change).

The stabilizer generators change as:

```
IIIXXXX IIIIIII -> IIIXXXX IIIXXXX = S1 T1
IXXIIXX IIIIIII -> IXXIIXX IXXIIXX = S2 T2
XIXIXIX IIIIIII -> XIXIXIX XIXIXIX = S3 T3
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```
IIIZZZZ IIIIIII -> IIIZZZZ IIIIIII = S4
IZZIIZZ IIIIIII -> IZZIIZZ IIIIIII = S5
ZIZIZIZ IIIIIII -> ZIZIZIZ IIIIIII = S6
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```
IIIIIII IIIXXXX -> IIIIIII IIIXXXX = T1
IIIIIII IXXIIXX -> IIIIIII IXXIIXX = T2
IIIIIII XIXIXIX -> IIIIIII XIXIXIX = T3
```

```
IIIIIII IIIZZZZ -> IIIZZZZ IIIZZZZ = S4 T4
IIIIIII IZZIIZZ -> IZZIIZZ IZZIIZZ = S5 T5
IIIIIII ZIZIZIZ -> ZIZIZIZ ZIZIZIZ = S6 T6
```

The S1 T1 means a multiplication of 2 14-qubit Pauli operations,  
not a tensor product of 2 7-qubit Pauli operations.

So, the stabilizer is the same as the one generated by S1-S6, T1-T6.