Open Problems

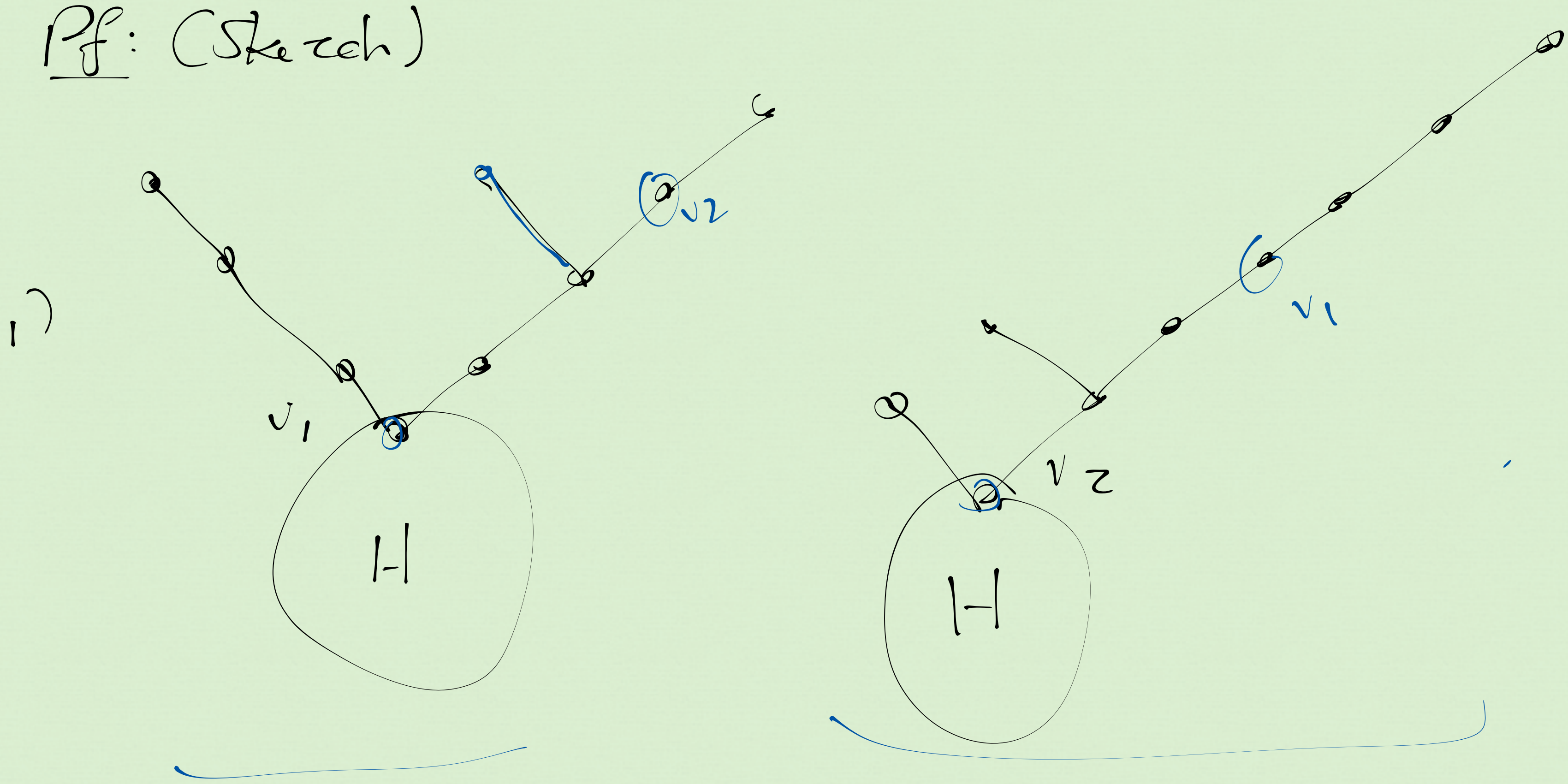
1. Cellular algebras of Schwenk Trees

Thm (Schwenk)

Almost all trees have a cospectral mate

Pf: (Sketch)

1)



are cospectral, $\forall$ H. tree

2) almost every tree has one of these as a "limb".

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Let X_1, X_2 be Schwenk's tree with H at v_1, v_2, resp.

Let $\mathcal{M}_1$, $\mathcal{M}_2$ be the cellular algebra of X_1, X_2 resp.

Is there a weak iso. $\phi : \mathcal{M}_1 \longrightarrow \mathcal{M}_2$ s.t.

$\phi(A(X_1)) = A(X_2)$?

Note: Is it true when H is $\bullet$ (hence, these two varieties are strongly cospectral)?

2. extensions of DRGs. & Terwilliger algebras.

X, Y are DRGs with vxs

$x_1, \dots, x_n$ and $y_1, \dots, y_n$.

s.t. $T(x_i) \cong T(y_i)$ as algebras and

$\exists$ iso which sents $A_i(X) \to A_i(Y)$

and $E_j^*(X, x_i) \to E_j^*(Y, y_i)$.

Is there a weak iso $\hat{W}_2(X) \to \hat{W}_2(Y)$?

What about $\hat{W}_3(X) \to \hat{W}_3(Y)$?

3. Jaeger algebras X, X'

If $J_3 \cong J_3' \Rightarrow \hat{W}_2(X) \xrightarrow[\text{iso}]{\text{weak}} \hat{W}_2(Y)$?

$J_4 \cong J_4' \Rightarrow \hat{W}_3(X) \xrightarrow[\text{iso}]{\text{weak}} \hat{W}_3(Y)$?

4. Does there exist two Latin square graphs X, Y

$\hat{W}_2(X) \xrightarrow[\text{iso}]{\text{weak}} \hat{W}_2(Y)$?

Note: by Bose, if a connected regular graph is cospectral to a Latin square graph of order $n > 23$, then it is itself a Latin square graph.

main classes not isotopy.

For $n=6$, there are 12 main iso. classes of Latin square and 32 536 Latin square "imposters".

For $n=7$, unknown.

there are 147 LSs.

For $n=8$, there are 283 657 LSs.

More Broad problems

Open problem: is there a DRG of large diameter with no non-trivial automorphisms?

Question X, Y with $\hat{W}_2(X) \ncong \hat{W}_2(Y)$, how do we find an easy certificate?

$\exists\ M(X)$ s.t. $M(X) \longmapsto M(Y)$ for every weak iso in $\hat{W}_k$? ←

Open problem

$\hat{W}_2$ for edge-distance regular graphs.

Question: higher "symmetry"

$\mathcal{P}$ is a class of graphs

X is locally $\mathcal{P}$-homogeneous if whenever $U \subseteq V(X)$ and $X[U] \in \mathcal{P}$, then each aut. of U extends to an aut. of X.

$\hat{W}_2$ of $\mathcal{P}$-homogeneous graphs

Question: higher transitivity

n-transitive: whenever 2 n-tuples are isometric $\Rightarrow$ there is an aut taking the first to the second.

distance-transitive = 2-transitive.

Thm (Cameron) 6-transitive graph $\Rightarrow$

complete multipartite, complete bipartite with a matching deleted or a cycle or 3 special graphs on 9, 12 and 20 vxs

9 ↳ Paley, 12 ⟶ icosahedron

$\Rightarrow$ n-trans $\forall n$.

For these, find Schur bases of $\hat{W}_k$ $k = 2, 3, \dots$?

20 vx graph: vxs 3-subsets of $[6]$

$a \sim b$ if $|a \cap b| = 2$. $\underbrace{J(6,3,2)}$