Problems

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Problem 1

Prove that the point graph of a generalized quadrangle of order (s, t) is strongly regular and give its parameters and eigenvalues in terms of s and t.

Problem 2

We consider a simple graph X (with no loops or parallel edges) on n vertices. Show the following:

(i) The adjacency matrix of X has exactly 1 distinct eigenvalues if and only if X has no edges.

(ii) The adjacency matrix of X has exactly 2 distinct eigenvalues if and only if X is a complete graph.

(iii) If X is vertex-transitive and has n distinct eigenvalues, then X is isomorphic to K_1 or K_2.

(iv) If X is regular with exactly three distinct eigenvalues, then X is strongly regular.

Problem 3

The folded cube is obtained from the cube by identifying antipodal vertices. Show that the folded cube is distance-regular and find the intersection array.

Problem 4

The Petersen graph is a strongly regular graph with parameters $(10, 3, 0, 1)$. Show that if Y is also a strongly regular graph with parameters $(10, 3, 0, 1)$, then Y is isomorphic to X.

Problem 5

Recently, Gritsenko found strongly regular graphs with parameters $(65, 32, 15, 16)$, a parameter class that was previously open. Show that any such that is not vertex-transitive. (Hint 1: consider the Sylow 5- and 13-subgroups of the automorphism group and the orbit partition of the graph induced by the action of these Sylow p-subgroups. Hint 2: some computation is required to compute the proof).

Problem 6

Let W and W' be two cellular algebras. Suppose $\phi : W \to W'$ respecting addition, multiplication, scalar multiplication and Schur multiplication; that is, for $A, B \in W$ and $c \in \mathbb{R}$,

(a) $\phi(AB) = \phi(A)\phi(B)$,

(b) $\phi(A \circ B) = \phi(A) \circ \phi(B)$,

(c) $\phi(A + B) = \phi(A) + \phi(B)$,

(d) $\phi(cA) = c\phi(A)$.

Show that ϕ is a weak isomorphism from W to W'.

Problem 7

Let X be a bipartite graph with bipartite colour classes V_0 and V_1. A *halved graph* of X is a graph whose vertices are one of the bipartite colour classes, and two vertices are adjacent if they are at distance 2 in X. Show that if X is a bipartite distance-regular graph and Y is a halved graph of X, then Y is distance-regular.