Errata for Algorithmic Game Theory

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Forward

- page xiii, line 7. modify punctuation as follows: " ...and its software. *And its algorithms:* Von Neumann..."

Chapter 1

- Page 3, line -6: "Prisoners' dilemma" should be "Prisoner's Dilemma"
- Page 14, line -8 (Example 1.9): While this example indeed has no pure equilibria, it does have a mixed-strategy equilibrium in which each player offers a price drawn from the probability distribution with support $[\frac{1}{2}, 1]$ and distribution function $F(x) = 2 - (1/x)$.
- Page 26, line 7: "Prisoners' Dilemma" should be "Prisoner's Dilemma"

Chapter 2

- All instances of the word "Nash" should be plain text, except for the following (listed by (page, line) numbers), which should remain in small caps: 30, 4; 31, -1 and -2; 32, lines 1, 6, 7, 13, 17, -6, and -2; 33, -3; 34, lines 7, 20, and 21 (see also below for revised paragraph); 37, lines -4 and -2; 38, -2; 39, -17; 41, 20; 42, 4; and 49, -6.
- Page 34, the 5th paragraph should be replaced with the following: "Incidentally, the problem of finding *any* Nash equilibrium in a symmetric game is also equivalent to NASH (and in fact via the same reduction above, can you see why?). But how hard is it to find a *nonsymmetric* Nash equilibrium in a symmetric game? It cannot be easier than NASH (can you see why the same reduction above proves this?) but it could be harder — for example, it could be NP-complete."

Chapter 5

- Page 103, end of second paragraph of Section 5.1, footnote with the following text should be added: "The First Welfare Theorem should come as a big surprise. On the one hand, a competitive equilibrium is arrived at when all agents actively pursue their self-interests (in a decentralized manner). On the other hand, the notion of Pareto optimality is an inherently centralized notion – it seeks maximal benefit for the society as a whole. The following quote from Adam Smith's classic work, *The Wealth of Nations,* 1776, is most illuminating: "It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest." ... Each participant in a competitive economy is "led by an invisible hand to promote an end which was no part of his intention." Observe that the First Welfare Theorem is essentially a testament to the power of pricing mechanisms."

- Page 104, end of line 8, footnote with the following text should be added: "See Shoven and Whalley (1992) for extensive discussion on the area of applied general equilibrium analysis, which aims at using insights from the Walrasian general equilibrium theory to developing realistic models of actual economies. As an instance of the use of such a model (to study the impact of NAFTA), see Kehoe and Kehoe (1994)."

Chapter 9

- Page 219, section 9.3.4, definition 9.19: In the three places that "v_i" appears, the subscript "i" should be changed to "j".

- Page 220, last line of the proof at the top of the page: the two occurrences of "v_i" should be changed to "v_j".

Chapter 10

- Definition 10.5 (Top Trading Cycles Algorithm) should read as follows: "Construct a directed graph with one vertex for each agent. Insert a directed edge from i to j if house j is agent i's most-preferred one. An edge of the form (i, i) will be called a loop. First identify all directed cycles and loops of this graph. Because preferences are strict and the outdegree of each vertex is exactly one, the set of such cycles and loops is both non-empty and node disjoint. Let N_1 be the set of vertices (agents) incident to these cycles. Each cycle implies a sequence of swaps. For example, suppose $i_1 \rightarrow i_2 \rightarrow i_3 \rightarrow \ldots \rightarrow i_r$ is such a cycle. Give house i_1 to agent i_r, house i_r to agent i_{r-1} and so on.

  After all such swaps are performed, create a new graph with one vertex for each agent in $N \setminus N_1$. Insert a directed edge from i to j if house j is agent i's most-preferred house among those owned by agents in $N \setminus N_1$. Let N_2 be the set of vertices (agents) incident to the loops and cycles in this graph, and let these agents swap houses as we did with N_1.

  Form N_3, etc., similarly. The Top Trading Cycle Algorithm (TTCA) yields the resulting matching."

- The proof of Theorem 10.13 should read as follows: "Suppose there is a profile of preferences $\pi = (\succ_{m_1}, \succ_{m_2}, \ldots, \succ_{m_n})$ for the men, such that man m_1, say, can misreport his preferences and obtain a better match. To express this formally, let μ be the stable matching obtained by applying the male-proposal algorithm to the profile π. Let ν be the stable matching that results under the male-proposal algorithm when m_1 reports the preference ordering $\succ_*$ instead, i.e. applied to the profile $\pi^1 = (\succ_*, \succ_{m_2}, \ldots, \succ_{m_n})$. We show that if $\nu(m_1) \succ_{m_1} \mu(m_1)$, then ν is not stable with respect to π_1, which is a contradiction. For notational convenience we write $a \succeq_m b$ to mean $[a \succ_m b \text{ or } a = b]$.

Let $R = \{m : \nu(m) \succ_m \mu(m)\}$. We show that for any $m \in R$ and $w = \nu(m)$, $m' \equiv \mu(w) \in R$. If $m' = m_1$, we are done. Otherwise, since $w \succ_m \mu(m)$, stability of μ implies $m' \succ_w m$. Stability of ν (for π_1) then implies $\nu(m') \succ_{m'} w$. Therefore $m' \in R$, and we can define $S = \{w : \nu(w) \in R\} = \{w : \mu(w) \in R\}$.

Since $\mu(w) \succ_w \nu(w)$ for any $w \in S$, during execution of the male-proposal algorithm on π, each $w \in S$ rejects $\nu(w) \in R$ at some iteration. Let m be the last man in R to make a proposal during the execution of the male-proposal algorithm. This proposal is made to $w = \mu(m) \in S$ who, by choice of m, must have rejected $\nu(w)$ at some strictly earlier iteration of the algorithm. This means that when m proposes to w, she must reject an outstanding proposal from some $m' \notin R$ such that $m' \succ_w \nu(w)$. Since $m' \notin R$, we have $w \succ_{m'} \mu(m') \succeq_{m'} \nu(m')$. Hence (m', w) form a blocking pair for ν at π^1 (since $m' \neq m_1$)."

Chapter 11

- Page 299, exercise 11.9(a), second bullet: "argmax" should be "argmin" (the subscript k remains).

Chapter 24

- In the first paragraph on page 616, the last sentence should be changed from

  "As a result, in time step $t = 2$, nodes 1 and -1 will switch back to behavior A, and the new behavior will have died out completely."

  to

  "The system will continue oscillating between even-numbered and odd-numbered nodes adopting B, but no node will ever permanently adopt B."

Chapter 28

- The proof of Theorem 28.2 should read as follows: "Order the bidders so that $v_1 \geq v_2 \geq \ldots \geq v_n$. Let p_i^* be the Vickrey price of slot i. Let bidder 1 bid $b_1 = v_1$ and each bidder $j \geq 2$ bids $b_j = \frac{p_{j-1}^*}{\mu_{j-1}}$. First we show that under the rules of the GSP, bidder 1 is assigned to slot 1, bidder 2 to slot 2, and so on. To do this it suffices to show that $b_{j-1} \geq b_j$. Since the optimal assignment is locally envy-free we have

$$\mu_j v_j - p_j^* \geq \mu_{j-1} v_j - p_{j-1}^*.$$

  Therefore

$$\mu_j[v_j - (p_j)/(\mu_j))] \geq \mu_{j-1}[v_j - (p_{j-1})/(\mu_{j-1})] \geq \mu_j[v_j - (p_{j-1})/(\mu_{j-1})]$$
$$\Rightarrow [v_j - (p_j)/(\mu_j))] \geq [v_j - (p_{j-1})/(\mu_{j-1})] \Rightarrow (p_{j-1})/(\mu_{j-1}) \geq (p_j)/(\mu_j)).$$

  Which implies $b_{j-1} \geq b_j$.

  Hence if each bidder j bids b_j the GSP returns the optimal assignment. It is also easy to see that bidder $j \leq m$ pays p_j^* for their slot. Bidder $j > m$ pays zero. Since each bidder pays their Vickrey price and receives the slot they would have under the efficient allocation, no bidder has a unilateral incentive to change their bid. Therefore we have an equilibrium that, from Theorem 1, is envy-free."