ACTSC 232 - Winter 2018 - Tutorial 1

First (or preferred) Name: SOLUTIONS   Last (family) Name: ____________

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1. In the TV commercials for "Guaranteed Life Plus", they say "if you're between 50 and 75, you cannot be turned down, we won't ask you any health questions, and no sales person will visit." How can they offer this type of simplified issue policy? Would you purchase it?

The insurer assumes policyholders are in poor health and charges a high premium to cover the risk. If I was quite sick I might buy this policy, but if I was healthy I could probably get a better rate from a different company.

2. Think Like an Actuary: if your company offers both life insurance (payable on policyholder's death) and life annuities (annual benefit payable while policyholder is alive), which policyholders would you require to provide stronger evidence of good health? Why?

The insurance policyholders.
From the insurer's perspective, an unhealthy insurance policyholder can lose a lot of money if they die earlier than expected. But for an annuity policyholder, the insurer saves money if they die early

3. Given $S_0(x) = \frac{1}{1+x}$, $x \geq 0$, find expressions for the following, simplifying as far as possible:

(a) $F_0(x) = 1 - S_0(x) = 1 - \frac{1}{1+x} = \frac{x}{1+x}, \quad x \geq 0.$

(b) $f_0(x) = F_0'(x)$ or $-S_0'(x) = \frac{1}{(1+x)^2}, \quad x \geq 0.$

(c) $S_x(t) = \dfrac{S_0(x+t)}{S_0(x)} = \dfrac{\frac{1}{1+x+t}}{\frac{1}{1+x}} = \dfrac{1+x}{1+x+t}, \quad t \geq 0.$

(d) Determine whether $S_0(x)$ satisfies all 6 conditions for a survival model.

1. $S_0(0) = \frac{1}{1+0} = 1$ ✓
2. $\lim_{x\to\infty} S_0(x) = \lim_{x\to\infty} \frac{1}{1+x} = 0$ ✓
3. $S_0'(x) = -\frac{1}{(1-x)^2} < 0$ so non-increasing ✓
4. differentiable ✓
5. $\lim_{x\to\infty} x\, S_0(x) = \lim_{x\to\infty} \frac{x}{1+x} = 1 \neq 0$ not satisfied!
6. $\lim_{x\to\infty} x^2 S_0(x) = \lim_{x\to\infty} \frac{x^2}{1+x} = \infty \neq 0$ not satisfied!

the model is valid but not very realistic

ACTSC 232 - WINTER 2018 - TUTORIAL 2

First (or preferred) Name: SOLUTIONS Last (family) Name: ______________

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1. Determine ${}_{10}p_{20}$ if $\mu_x = 0.01e^{0.001x}$ for all $x > 0$

$${}_{10}p_{20} = e^{-\int_0^{10} 0.01 e^{0.001(20+r)}dr} = e^{-\frac{0.01}{0.001}\left[e^{0.001(20+r)}\right]_0^{10}}$$

$$= e^{-10(e^{0.03} - e^{0.02})} = 0.90255$$

2. Using a modified De Moivre model ${}_tp_x = \left(1 - \frac{t}{120-x}\right)^{\frac{1}{6}}$, find q_{20} and q_{110}. Approximate these quantities by $\mu_{20.5}$ and $\mu_{110.5}$, respectively. Why is one less accurate than the other?

$$q_{20} = 1 - p_{20} = 1 - \left(1 - \frac{1}{120-20}\right)^{1/6} = 0.001674$$

$$q_{110} = 1 - p_{110} = 1 - \left(1 - \frac{1}{120-110}\right)^{1/6} = 0.017407$$

from class we know $\mu_x = \frac{1}{6(120-x)}$ for this model (or derive it

$\mu_{20.5} = 0.001675$, $\mu_{110.5} = 0.017544$

The older age is less accurate since μ_x is steeply increasing

3. Given $p_x = 0.99$, $p_{x+1} = 0.985$, ${}_3p_{x+1} = 0.95$, $q_{x+3} = 0.02$, calculate to 5 decimal places:

(a) $p_{x+3} = 1 - q_{x+3} = 0.98$

(b) ${}_2p_x = (p_x)(p_{x+1}) = (0.99)(0.985) = 0.97515$

(c) ${}_2p_{x+1}$ we know ${}_3p_{x+1} = ({}_2p_{x+1})(p_{x+3})$

so ${}_2p_{x+1} = \frac{{}_3p_{x+1}}{p_{x+3}} = \frac{0.95}{0.98} = 0.96938$

(d) ${}_3p_x$

$= (p_x)({}_2p_{x+1}) = (0.99)(0.96938) = 0.95969$

(e) ${}_{1|2}q_x = (p_x)({}_2q_{x+1}) = (0.99)(1 - 0.96938) = 0.03031$

or $p_x - {}_3p_x = 0.99 - 0.95969 = 0.03031$

4. Think Like an Actuary: discuss some advantages and disadvantages of using a simple mortality model (e.g. De Moivre, Gompertz, etc) rather than the actual human mortality curve.

adv: easy to use and explain, few parameters, nice mathematical results.

dis: doesn't capture features of actual mortality, results may not be accurate.

ACTSC 232 - Winter 2018 - Tutorial 3

First (or preferred) Name: SOLUTIONS Last (family) Name: ____________

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1. A life's mortality is ${}_tp_x = (1 - \frac{t}{100-x})^{0.5}$ for $x \le 100$, $t \le 100 - x$, and 0 for $t \ge 100 - x$.

   (a) Calculate μ_x. $f_0(x) = -0.5(1 - \frac{x}{100})^{-0.5}(-\frac{1}{100}) = \frac{0.5/100}{(1-\frac{x}{100})^{0.5}}$ $S_0(x) = (1 - \frac{x}{100})^{0.5}$

   so $\mu_x = \frac{0.5/100}{(1-\frac{x}{100})} = \frac{1}{200-2x}$

   (b) Calculate $\mathring{e}_{45}$.

   $\mathring{e}_{45} = \int_0^{55} (1 - \frac{t}{55})^{0.5} dt = \left[\frac{-55}{1.5}(1-\frac{t}{55})^{1.5}\right]_0^{55} = \frac{55}{1.5} = \boxed{36.67}$

   (c) Calculate $\mathrm{Var}(T_{45})$. $E[T_{45}^2] = 2\int_0^{55} t(1-\frac{t}{55})^{0.5} dt = 2 \times 55^2(\frac{2}{3} - \frac{2}{5}) = 1613.33$

   so $\mathrm{Var}(T_{45}) = 1613.33 - (36.67)^2 = \boxed{268.89}$

2. For a life age 20, it is estimated that the impact of self-driving cars will be an increase of 5 years in $\mathring{e}_{20}$. If currently ${}_tp_x = 1 - \frac{t}{110-x}$ (i.e. De Moivre with $\omega = 110$) and after self-driving cars, ${}_tp_x = 1 - \frac{t}{\omega^*-x}$, then find ω^*, the new limiting age thanks to self-driving cars.

   $T_{20} \sim U(0, 110-20) = U(0,90)$ since De Moivre

   so $\mathring{e}_{20} = 45$ in current model.

   new $\mathring{e}_{20} = 45 + 5 = 50$

   so we need ω^* such that $T_{20}^* \sim U(0, \omega^*-20)$ has average 50.

   that means $\omega^* - 20 = 100$ and thus $\boxed{\omega^* = 120}$

3. Think Like an Actuary: briefly discuss what effects (both positive and negative) self-driving cars might have on the insurance industry. lots of great ideas, nice work!

   positive: • less risk of accident and death → lower claims
   • reduce "accident bump" in mortality

   negative: • if all cars use similar software, could be a major security risk that affects large numbers of policyholders
   • premiums will face pressure to be lower too

4. Using the Illustrative Life Table (just handed out), write the actuarial notation and calculate the probabilities (to 5 decimals) that a life age 40:

   (a) dies before reaching age 42. ${}_2q_{40} = \frac{l_{40} - l_{42}}{l_{40}} = 0.00575$

   (b) dies between age 41 and 42. ${}_{1|1}q_{40} = \frac{d_{41}}{l_{40}}$ or $\frac{l_{41} - l_{42}}{l_{40}} = 0.00297$

   (c) survives to age 100. ${}_{60}p_{40} = \frac{l_{100}}{l_{40}} = 0.00430$

   (d) dies between age 98 and 100. ${}_{58|2}q_{40} = \frac{l_{98} - l_{100}}{l_{40}}$ or $\frac{d_{98} + d_{99}}{l_{40}} = 0.00643$

ACTSC 232 - WINTER 2018 - TUTORIAL 4

First (or preferred) Name: SOLUTIONS Last (family) Name: __________

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1. You are given the following extract from a life table:

x	45	46	47	48
l_x	99034	98958	98875	98784

For each of the following, write the probability in words and calculate the value to 5 decimal places using both UDD and CFM.

(a) ${}_{0.7}p_{46}$ the probability a life age 46 survives 0.7 years

UDD: $1 - {}_{0.7}q_{46} = 1 - 0.7q_{46} = 1 - 0.7\left(\frac{l_{46} - l_{47}}{l_{46}}\right) = 0.99941$

CFM: $(p_{46})^{0.7} = \left(\frac{l_{47}}{l_{46}}\right)^{0.7} = 0.99941$

(b) ${}_{0.3}p_{47.2}$ the probability a life age 47.2 survives 0.3 years

UDD: $\frac{{}_{0.5}p_{47}}{{}_{0.2}p_{47}} = \frac{1 - 0.5q_{47}}{1 - 0.2q_{47}} = \frac{1 - 0.5\left(\frac{l_{47} - l_{48}}{l_{47}}\right)}{1 - 0.2\left(\frac{l_{47} - l_{48}}{l_{47}}\right)} = 0.99972$

CFM: $(p_{47})^{0.3} = \left(\frac{l_{48}}{l_{47}}\right)^{0.3} = 0.99972$

(c) ${}_{1.7}q_{45.5}$ the prob. a life age 45.5 dies within 1.7 years

UDD: $1 - \frac{{}_{2.2}p_{45}}{{}_{0.5}p_{45}} = 1 - \frac{(0.8l_{47} + 0.2l_{48})/l_{45}}{(0.5l_{45} + 0.5l_{46})/l_{45}} = 0.00141$

CFM: $1 - ({}_{0.5}p_{45.5})(p_{46})({}_{0.2}p_{47}) = 1 - (p_{45})^{0.5}p_{46}(p_{47})^{0.2} = 0.00141$

2. Think Like an Actuary: The advent of relatively cheap genetic testing (e.g. 23andme, AncestryDNA, etc) gives people information about their genetic predisposition for certain diseases.

Come up with as many arguments as you can to support the position that insurance companies should (NOT) be allowed to require genetic testing results from policyholders.

SHOULD	SHOULD NOT
• more accurate info about risk of policyholders	• might unfairly discriminate against some policyholders
• if policyholders have info and insurers don't, people at higher risk may buy more comprehensive insurance coverage, experience worse claims, making the insurer raise premiums for everyone → "antiselection"	• genetic test results are not always a guarantee of higher risk
	• privacy concerns - if there is a data security breach, very sensitive info could be leaked
	• currently against Canadian law (Bill S-201: Genetic Non-Discrimination Act)

ACTSC 232 - Winter 2018 - Tutorial 5

First (or preferred) Name: ________________ Last (family) Name: ________________

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This tutorial is review of the material for Test 1 next week.

1. Define the following terms in as much detail as possible: (try without looking at your notes)

   - survival function

     the probability a life age x survives at least t years

     $S_x(t) = \frac{S_0(x+t)}{S_0(x)}$ also called ${}_tp_x$

   - force of mortality

     the instantaneous rate of death at age x (multiplier for short interval length)

     $\mu_x = \frac{f_0(x)}{S_0(x)}$ $\mu_{x+t} = \frac{f_x(t)}{S_x(t)}$

   - complete expectation of life

     the mean future lifetime of (x)

     $\mathring{e}_x = E[T_x] = \int_0^\infty {}_tp_x \, dt$

   - curtate future lifetime random variable

     the number of full years of future lifetime for (x)

     $K_x = \lfloor T_x \rfloor$ a discrete r.v. with p.f. $P(K_x = k) = {}_{k|}q_x$

   - selection period

     the time past which the selection effect (improved mortality during first years of policy) wears off and ultimate mort applies

     D when $d \geq D$, $\mu_{[x]+d} = \mu_{x+d}$

   - mortality improvement

     due to increases in technology, medicine, standard of living

     can model with mortality reduction factor r_x

   - whole life insurance

     a policy where a benefit is payable on policyholder's death

     can be paid immediately, at end of year, or at end of $\frac{1}{m}$-year

   - present value random variable

     the random variable describing the present value of the benefit payment

     $Z = v^{T_x} = e^{-\delta T_x}$ for whole life

     $E[Z] = \bar{A}_x = \int_0^\infty e^{-\delta t} \, {}_tp_x \mu_{x+t} \, dt$

2. You are given the following extract from a select-and-ultimate life table with $D = 4$:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	l_{x+4}	$x+4$
[40]	100,000	99,899	99,724	99,520	99,288	44
[41]	99,802	99,689	99,502	99,283	99,033	45
[42]	99,597	99,471	99,268	99,030	98,752	46

Calculate ${}_{0.4|2.1}q_{[40]+2.6}$ to 5 decimal places using both UDD and CFM.

UDD: $${}_{0.4|2.1}q_{[40]+2.6} = \frac{l_{[40]+2.6+0.4} - l_{[40]+2.6+0.4+2.1}}{l_{[40]+2.6}}$$

$$= \frac{l_{[40]+3} - l_{45.1}}{l_{[40]+2.6}} = \frac{l_{[40]+3} - (0.9\,l_{45} + 0.1\,l_{46})}{0.4\,l_{[40]+2} + 0.6\,l_{[40]+3}}$$

$$= 0.00517160$$

or $${}_{0.4}p_{[40]+2.6}\ {}_{2.1}q_{[40]+3} = \frac{p_{[40]+2}}{{}_{0.6}p_{[40]+2}}\left(1 - {}_{2}p_{[40]+3}\ {}_{0.1}p_{45}\right)$$

$$= \frac{l_{[40]+3}/l_{[40]+2}}{1 - 0.6\,q_{[40]+2}}\left(1 - \frac{l_{45}}{l_{[40]+3}}\left(1 - 0.1\,q_{45}\right)\right)$$

$$= 0.00517160$$

CFM:

$${}_{0.4}p_{[40]+2.6}\ {}_{2.1}q_{[40]+3}$$

$$= \left(p_{[40]+2}\right)^{0.4}\left(1 - {}_{2}p_{[40]+3}\left(p_{45}\right)^{0.1}\right)$$

$$= \left(\frac{l_{[40]+3}}{l_{[40]+2}}\right)^{0.4}\left(1 - \frac{l_{45}}{l_{[40]+3}}\left(\frac{l_{46}}{l_{45}}\right)^{0.1}\right) = 0.00517197$$

3. Calculate the EPV of an insurance which pays \$1 immediately on the death of a life age x, if the life follows a De Moivre model with $\omega = 100$, and $\delta = 0.05$. Also calculate the variance of the present value. Are these increasing/decreasing/constant functions of x? Is that realistic?

Note for De Moivre, ${}_tp_x = \frac{100-x-t}{100-x}$, $\mu_{x+t} = \frac{1}{100-x-t}$

so ${}_tp_x\,\mu_{x+t} = \frac{1}{100-x}$ (the pdf is constant)

$$\bar{A}_x = \int_0^{100-x} e^{-\delta t}\,{}_tp_x\,\mu_{x+t}\,dt = \frac{1}{100-x}\int_0^{100-x} e^{-0.05t}\,dt = \frac{1}{100-x}\left[\frac{-1}{0.05}e^{-0.05t}\right]_0^{100-x}$$

$$= \frac{1-e^{-0.05(100-x)}}{0.05(100-x)}$$

this is an increasing function of x, which makes sense because an older life will tend to die sooner, making the payment have ↑ PV

${}^2\bar{A}_x$ is just $\bar{A}_x$ evaluated at $2\delta = 0.10$, or $\frac{1-e^{-0.10(100-x)}}{0.10(100-x)}$

$$\text{so } Var(Z) = \frac{1-e^{-0.10(100-x)}}{0.10(100-x)} - \left(\frac{1-e^{-0.05(100-x)}}{0.05(100-x)}\right)^2$$

this function decreases to 0 as $x \to 100$, since there is less uncertainty for older ages. (It actually increases for age $0 \to 35$ though)

ACTSC 232 - Winter 2018 - Tutorial 6

First (given) Name: SOLUTIONS Last (family) Name: ______________

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1. Describe in words the benefits payable under the insurances with the present values given below. Also write down $E[Z_1]$ and $E[Z_2]$ using actuarial functions.

   (a) $Z_1 = \begin{cases} 20000v^{K_x^{(12)}+\frac{1}{12}}, & K_x^{(12)} \leq 14\frac{11}{12} \\ 10000v^{15}, & K_x^{(12)} \geq 15 \end{cases}$

   Pays \$20,000 at the end of the month of death within 15 years, or \$10,000 on survival to 15 years

   $E[Z_1] = 20{,}000 A^{(12)\,1}_{x:\overline{15}|} + 10{,}000\, A_{x:\overline{15}|}^{\ \ 1}$

   or $20{,}000 A^{(12)}_{x:\overline{15}|} - 10{,}000\, A_{x:\overline{15}|}^{\ \ 1}$

   (b) $Z_2 = \begin{cases} 10000(K_x+1)v^{K_x+1}, & K_x \leq 9 \\ 0, & K_x \geq 10 \end{cases}$

   Pays an arithmetically increasing benefit of \$10,000(K_x+1) at the end of the year of death, if death occurs within 10 years.

   $E[Z_2] = 10{,}000 (IA)^1_{x:\overline{10}|}$

2. Given $\mu_{x+t} = \mu$ for all $t \geq 0$, derive expressions (and simplify as far as possible) for the following functions, assuming a constant force of interest δ.

   note: ${}_tp_x = e^{-\int_0^t \mu_{x+r}\,dr} = e^{-\mu t}$

   (a) $\bar{A}_x = \int_0^\infty e^{-\delta t}\, {}_tp_x \mu_{x+t}\, dt$

   $= \int_0^\infty e^{-\delta t} e^{-\mu t} \mu\, dt = \frac{-\mu}{\delta+\mu} e^{-(\delta+\mu)t}\Big|_0^\infty = \frac{\mu}{\delta+\mu}$

   (b) $\bar{A}^1_{x:\overline{n}|} = \int_0^n e^{-\delta t}\, {}_tp_x \mu_{x+t}\, dt$

   $= \frac{-\mu}{\delta+\mu} e^{-(\delta+\mu)t}\Big|_0^n = \frac{\mu}{\delta+\mu}\left(1 - e^{-(\delta+\mu)n}\right)$

   (c) $A_{x:\overline{n}|}^{\ \ 1}$

   $= v^n\, {}_np_x$

   $= e^{-\delta n} e^{-\mu n} = e^{-(\delta+\mu)n}$

3. Think Like an Actuary:
   Without doing any calculations, would the covariance between pairs of benefits be positive, negative, or 0? Fill in the following chart with "+", "−", or "0"

principle: Consider whether a short lifetime would be better or worse for the insurer for both products.

covariance	whole ins	term ins	pure end	end ins	def whole ins	whole annuity
whole ins	+					
term ins	+	+				
pure end	−	−	+			
end ins	+	+	−	+		
def whole ins	−	−	+	−	+	
whole annuity	−	−	+	−	+	+

Why might it be useful for an insurer to offer pairs of products with negative covariance?

So they can diversify their risk. If there was a systematic improvement (or a catastrophe/epidemic) that affected the mortality of many lives, the insurer would have less risk overall if some products and some

ACTSC 232 - WINTER 2018 - TUTORIAL 7

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1. Describe in words the annuity with the PVRV given (try without consulting your notes) and write down the actuarial notation for the expected present value.

$$Y = \begin{cases} \ddot{a}^{(4)}_{\overline{K_x^{(4)}+\frac{1}{4}}|}, & K_x^{(4)} \leq 19.75 \\ \ddot{a}^{(4)}_{\overline{20}|}, & K_x^{(4)} \geq 20 \end{cases}$$

$E[Y] = \ddot{a}^{(4)}_{x:\overline{20}|}$

Quarterly 20-year term annuity that pays 25¢ at the start of each $\frac{1}{4}$-year while (x) is alive, for a maximum of 20 years.

2. You are given: (i) The force of interest and force of mortality are constant and equal. ($\delta = \mu$) (ii) $\bar{a}_x = 12.50$. Calculate the following:

   (a) the force of interest

   $\bar{a}_x = \int_0^\infty e^{-\delta t} e^{-\mu t} dt = \frac{1}{\delta+\mu} = \frac{1}{2\delta} = 12.50 \quad \therefore \delta = 0.04$

   (b) $\bar{A}_x$

   $= \int_0^\infty e^{-\delta t} e^{-\mu t} \mu \, dt = \frac{\mu}{\delta+\mu} = 0.5$ or $\bar{A}_x = 1 - \delta \bar{a}_x = 0.5$

   (c) the standard deviation of $\bar{a}_{\overline{T_x}|}$

   ${}^2\bar{A}_x = \int_0^\infty e^{-2\delta} e^{-\mu t} \mu \, dt = \frac{\mu}{2\delta+\mu} = 0.333$

   so $Var(\bar{a}_{\overline{T_x}|}) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{0.333 - 0.5^2}{0.04^2} = 52.083$

   so $SD = 7.217$

3. Think Like an Actuary: A good skill to have is to be able to quickly judge your numerical answers using logic to see if they make sense. Without doing any calculations, indicate whether each of the following results make sense, and explain why or why not.

   (a) $\bar{A}^{1}_{20:\overline{20}|} < \bar{A}_{20:\overline{20}|}$ ✓ the endowment insurance includes the term insurance plus a pure endowment, so EPV ↑

   (b) $\ddot{a}_{20:\overline{20}|} < \ddot{a}_{20:\overline{10}|}$ ✗ the 10-year annuity only pays for 10 years max rather than 20, so EPV should be ↓

   (c) $\ddot{a}_{20:\overline{10}|} > 10$ ✗ the maximum paid is 10 \$1 installments, which has a PV of less than \$10 (assuming $i > 0$)

   (d) $\bar{A}_{20:\overline{20}|} < \bar{A}_{20:\overline{10}|}$ ✓ the 10-year endowment ins. will likely pay sooner than the 20-year, so EPV ↑

4. Using the Illustrative Life Table with 6% interest per year effective, calculate the expected present value and the standard deviation of an annuity of \$20,000 per year paid annually in advance for a maximum of 3 years, contingent on the survival of a life currently age 40.

3 methods for mean:

① $A_{40:\overline{3}|} = A_{40} - {}_3E_{40} A_{43} + {}_3E_{40} = A_{40} - {}_3E_{40}(A_{43} - 1)$

$= 0.1613242 - \frac{92299.23}{1.06^3 \, 93131.64}(0.1843271 - 1) = 0.8400579$

so $EPV = 20{,}000 \left(\frac{1 - A_{40:\overline{3}|}}{d} \right) = \boxed{56{,}512.96}$

② $\ddot{a}_{40:\overline{3}|} = 1 \times q_{40} + (1+v)\,{}_{1|}q_{40} + (1+v+v^2)\,{}_2p_{40} = 2.82565$

so $EPV = \boxed{56512.95}$

③ $\ddot{a}_{40:\overline{3}|} = 1 + v\,p_{40} + v^2\,{}_2p_{40} = 2.82565$ so $EPV = \boxed{56512.95}$

2 methods for SD:

① ${}^2A_{40:\overline{3}|} = {}^2A_{40} - {}^2_3E_{40}({}^2A_{43} - 1) = 0.7057342$ so $SD = \sqrt{\frac{{}^2A_{40:\overline{3}|} - A_{40:\overline{3}|}^2}{d^2}} = \boxed{2159.15}$

② $E[Y^2] = 1^2 q_{40} + (1+v)^2\,{}_{1|}q_{40} + (1+v+v^2)^2\,{}_2p_{40} = 7.995926$ so $SD = \boxed{2158.13}$

ACTSC 232 - Winter 2018 - Tutorial 8

First (or preferred) Name: SOLUTIONS Last (family) Name: ____________

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This tutorial is review of the material for Test 2 next week.

1. Define the following terms in as much detail as possible: (try without looking at your notes)

- deferred insurance pays on death only if after n years

$$Z = \begin{cases} 0 & T_x < n \\ v^{T_x} & T_x \geq n \end{cases} \qquad {}_{n|}\bar{A}_x = {}_nE_x \bar{A}_{x+n}$$

- pure endowment pays on survival to time n

$$Z = \begin{cases} 0 & T_x < n \\ v^n & T_x \geq n \end{cases} \qquad A_{x:\overline{n|}}^{\;\;1} = {}_nE_x = v^n {}_np_x$$

- increasing insurance benefit increases based on time of death

arithmetic $\rightarrow$ $(IA)_x$ or $(\bar{I}\bar{A})_x$ or $(I\bar{A})_x$

geometric $\rightarrow$ use modified interest rate $v^* = \frac{1+j}{1+i}$

- claims acceleration an alternative to UDD to approx $A_x^{(m)}$ or $\bar{A}_x$ based on correcting the PV for time of death

$$A_x^{(m)} \approx (1+i)^{\frac{m-1}{2m}} A_x \qquad \bar{A}_x \approx (1+i)^{0.5} A_x$$

- term annuity pays \$1 per year while (x) is alive, for $\overline{n|}$ max

$$Y = \begin{cases} \ddot{a}_{\overline{K_x+1|}} & K_x \leq n-1 \\ \ddot{a}_{\overline{n|}} & K_x \geq n \end{cases} \qquad \ddot{a}_{x:\overline{n|}} = \frac{1 - A_{x:\overline{n|}}}{d}$$

- guaranteed annuity pays \$1 per year for $\overline{n|}$, then until death

$$Y = \begin{cases} \ddot{a}_{\overline{n|}} & K_x \leq n-1 \\ \ddot{a}_{\overline{K_x+1|}} & K_x \geq n \end{cases} \qquad \ddot{a}_{\overline{x:\overline{n|}}} = \ddot{a}_{\overline{n|}} + {}_nE_x \ddot{a}_{x+n}$$

- Woolhouse approximation a way to approx $\ddot{a}_x^{(m)}$ or $\bar{a}_x$, more accurate than UDD

$$\ddot{a}_x^{(m)} = \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\delta + \mu_x)$$

$$\ddot{a}_{x:\overline{n|}}^{(m)} = \ddot{a}_{x:\overline{n|}} - \frac{m-1}{2m}(1 - {}_nE_x) - \frac{m^2-1}{12m^2}\left(\delta + \mu_x - {}_nE_x(\delta + \mu_{x+n})\right)$$

- accumulation

if \$1 deposited at the start of each year for n years (if alive), the share for each survivor at time n is

$$\ddot{s}_{x:\overline{n|}} = \frac{\ddot{a}_{x:\overline{n|}}}{{}_nE_x}$$

2. First cross out any items below that are not valid actuarial notation. Then rearrange the valid ones into order from smallest to largest within each group, assuming a positive interest rate. Explain your reasoning.

- $\ddot{a}_x^{(2)}$, $a_x^{(2)}$, $\ddot{a}_x^{(12)}$, $a_x^{(4)}$, $\cancel{\bar{a}_x^{(12)}}$, $\bar{a}_x$, a_x, ${}_{n|}a_x$

$${}_{n|}a_x < a_x < a_x^{(2)} < a_x^{(4)} < \bar{a}_x < \ddot{a}_x^{(12)} < \ddot{a}_x^{(2)}$$

no payments for first $\overline{n|}$

earlier payments have higher pvs (*) and more paid in 1st year of life

- $A_x^{(2)}$, $\bar{A}^1_{x:\overline{n|}}$, $\cancel{A_{x:\overline{n|}}}$, $A_{x:\overline{n|}}$, $(IA)_{x:\overline{n|}}$, A_x, $A^1_{x:\overline{n|}}$, $A^{(12)}_{x:\overline{n|}}$, $\cancel{A^{(4)1}_{x:\overline{n|}}}$.

$$A^1_{x:\overline{n|}} < \bar{A}^1_{x:\overline{n|}} < A_x < A_x^{(2)} < A_{x:\overline{n|}} < A^{(12)}_{x:\overline{n|}} < (IA)_{x:\overline{n|}}$$

cts pays sooner *

term may not pay at all

*

end. ins. pays sooner

*

increasing may pay > $1 benefit

- $\ddot{a}_{\overline{n|}}$, $\ddot{s}_{x:\overline{n|}}$, $a^{(12)}_{x:\overline{n|}}$, $\ddot{a}_{\overline{x:\overline{n|}}}$, $\bar{a}_{x:\overline{n|}}$, $\ddot{a}^{(4)}_{\overline{x:\overline{n|}}}$, $s^{(2)}_{x:\overline{n|}}$, $\cancel{\ddot{a}_{x\overline{n|}}}$

$$a^{(12)}_{x:\overline{n|}} < \bar{a}_{x:\overline{n|}} < \ddot{a}_{\overline{n|}} < \ddot{a}^{(4)}_{\overline{x:\overline{n|}}} < \ddot{a}_{\overline{x:\overline{n|}}} < s^{(2)}_{x:\overline{n|}} < \ddot{s}_{x:\overline{n|}}$$

*

term may not pay all n years

g'teed pays n years for sure

*

as long as n is reasonably long, acc. value > pv.

*

3. Using the Illustrative Life Table and 6% interest, calculate $\ddot{a}_{80:\overline{15|}}$ and estimate $a^{(4)}_{80:\overline{15|}}$ using all three approximations (UDD, Woolhouse, and simplified.) If you still have time left, also calculate the standard deviation of an annual 15-year term annuity immediate issued to (80).

$$\ddot{a}_{80:\overline{15|}} = \ddot{a}_{80} - v^{15}\frac{l_{95}}{l_{80}}\ddot{a}_{95} = 5.816444$$

UDD: $\ddot{a}^{(4)}_{80:\overline{15|}} = \alpha(4)\ddot{a}_{80:\overline{15|}} - \beta(4)\Big(1 - \underbrace{v^{15}\frac{l_{95}}{l_{80}}}_{{}_{15}E_{80}}\Big)$ where $\alpha(4) = \frac{id}{i^{(4)}d^{(4)}} = 1.000265$

$= 5.445954$ $\qquad \beta(4) = \frac{i - i^{(4)}}{i^{(4)}d^{(4)}} = 0.38424$

so $a^{(4)}_{80:\overline{15|}} = \ddot{a}^{(4)}_{80:\overline{15|}} - \frac{1}{4} + \frac{1}{4}v^{15}\frac{l_{95}}{l_{80}} = 5.203895$

Woolhouse: first estimate $\mu_{80} = 0.087713$ and $\mu_{95} = 0.347106$

$$\ddot{a}^{(4)}_{80:\overline{15|}} = \ddot{a}_{80:\overline{15|}} - \frac{3}{8}\left(1 - {}_{15}E_{80}\right) - \frac{15}{12 \times 16}\left(\ln(1.06) + \mu_{80} - {}_{15}E_{80}\left(\ln(1.06) + \mu_{95}\right)\right)$$

$= 5.442957$ so $a^{(4)}_{80:\overline{15|}} = 5.200898$

Simplified: $\ddot{a}^{(4)}_{80:\overline{15|}} = \ddot{a}_{80:\overline{15|}} - \frac{3}{8}\left(1 - {}_{15}E_{80}\right) = 5.453356$

So $a_{80:\overline{15|}} = 5.211297$

variance is $\dfrac{{}^2A_{80:\overline{15|}} - A_{80:\overline{15|}}^2}{d^2} = 8.547842$ so sd is 2.923669

ACTSC 232 - WINTER 2018 - TUTORIAL 9

First (or preferred) Name: SOLUTIONS Last (family) Name: ____________

ID #: ____________ UserID: ____________

1. L_0 is the loss random variable for a fully discrete 2-year term insurance of 1 issued to (x). The level annual premium is calculated using the equivalence principle. You are given: (i) $q_x = 0.1$, (ii) $q_{x+1} = 0.2$, and (iii) $v = 0.9$. Calculate $\mathrm{Var}(L_0)$.

First find P: $0.1v + 0.18v^2 = P(1 + 0.9v) \quad \therefore \quad P = 0.13028$

$$L_0 = \begin{cases} v - P & \text{with prob } q_x \\ v^2 - P(1+v) & \text{w prob } {}_{1|}q_x \\ -P(1+v) & \text{w prob } {}_2p_x \end{cases} \quad \text{so } L_0^2 = \begin{cases} 0.59247 & \text{w.p. } 0.1 \\ 0.31638 & \text{w.p. } 0.18 \\ 0.06127 & \text{w.p. } 0.72 \end{cases}$$

and $\mathrm{Var}(L_0) = E[L_0^2] - 0^2 = 0.16031$

2. Consider an annual premium 20-year deferred annuity issued to (40).
Benefits: Annuity of $70,000 per year paid monthly in advance, for a minimum of $\overline{5|}$.
If p/h dies in deferral period, return of premiums plus 2% interest plus $10,000, pd at EYOD.
Initial Expenses: 25% of first premium plus $150.
Renewal expenses during deferral: 2% of 2nd and subsequent premium plus $30.
Annuity expenses: $100 at end of deferral period, plus 0.5% of each annuity payment.
Basis: 6% int per year and Makeham mortality with A=0.0001, B=0.00035, c=1.075 gives:
$\ddot{a}_{40} = 13.521185$, $\ddot{a}_{60} = 9.596383$, $\ddot{a}_{65} = 8.455129$, ${}_{20}p_{40} = 0.751546$, ${}_5p_{60} = 0.850360$,
and for UDD approximation, use $\alpha(12) = 1.000281$ and $\beta(12) = 0.468120$

   (a) Find the EPV of the premiums (in terms of P).

   $P\ddot{a}_{40:\overline{20|}} = P(\ddot{a}_{40} - v^{20}\,{}_{20}p_{40}\,\ddot{a}_{60}) = 11.27241P$

   (b) Find the EPV of the fixed expenses (initial and renewal).

   $120 + 30\ddot{a}_{40:\overline{20|}} = 458.17233$

   (c) Find the EPV of the %-of-premium expenses (initial and renewal, in terms of P).

   $0.23P + 0.02P\ddot{a}_{40:\overline{20|}} = 0.45545P$

   (d) Find the EPV of the deferred annuity benefit, assuming UDD.

   $70{,}000\,{}_{20|}\ddot{a}^{(12)}_{\overline{40:\overline{5|}}} = 70{,}000\,{}_{20}E_{40}\left(\ddot{a}^{(12)}_{\overline{5|}} + {}_5E_{60}\ddot{a}^{(12)}_{65}\right) = 70{,}000v^{20}\,{}_{20}p_{40}\left[\frac{1-v^5}{d^{(12)}} + v^5\,{}_5p_{60}\left(\alpha(12)\ddot{a}_{65} - \beta(12)\right)\right] = 154599.7827$

   (e) Find the EPV of the annuity expenses.

   $100\,{}_{20}E_{40} + 0.005 \times$ result from (d) $= 796.43247$

   (f) Find the EPV of the death benefit. (The return of premium part has EPV $1.55402P$. You could find this using the spreadsheet if you had a computer.)

   $A^{1}_{40:\overline{20|}} = (1-d\ddot{a}_{40}) - {}_{20}E_{40}(1-d\ddot{a}_{60}) = 0.1276034$ so $1276.034 + 1.55402P$

   (g) Finally, write down the equation given by the equivalence principle (EPV benefits + EPV expenses = EPV premiums) and use it to solve for P. (a) = (b)+(c)+(d)+(e)+(f)

   $11.27241P = 458.17 + 0.45545P + 154599.78 + 796.43 + 1276.03 + 1.55402P$

   so $P = \dfrac{458.17 + 154599.78 + 796.43 + 1276.03}{11.27241 - 0.45545 - 1.55402} = \boxed{16{,}963.34}$

3. Think Like an Actuary
When selling insurance, it is common for a large commission to be paid by the insurer to the agent who sold the policy. Sometimes this amount can even be as much as 200% of the first year's premium. Name TWO different issues this practice could cause. (Hint: consider financial as well as ethical concerns.)

- New Business Strain: if costs are higher in the first year than the premium they take in, issuing a lot of policies (which take time to start making profits) can lose the company money.
- Moral Hazard: agents might recommend policies that get highest commissions rather than policies that suit their clients' needs.

ACTSC 232 - Winter 2018 - Tutorial 10

First (or preferred) Name: SOLUTIONS Last (family) Name: ________________

ID #: ________________ UserID: ________________

1. An impaired life age 35 is trying to purchase a fully discrete whole life insurance policy of \$100,000, with premiums payable for 30 years. They have two options:
   - Company A will give them a reduced sum insured according to a 4-year age rating, but charge the same premiums as a non-impaired life would pay.
   - Company B will give them the full sum insured, but charge higher premiums according to a constant addition to the force of mortality of 0.00939.

   Both companies use the Illustrative Life Table at 6% interest, and assume expenses of 20% of the first premium and 5% of all subsequent premiums.

   (a) Calculate the gross premium a non-impaired life would pay.

   need $P\ddot{a}_{35:\overline{30}|} = 100{,}000\,A_{35} + 0.15P + 0.05P\ddot{a}_{30:\overline{35}|}$

   where $\ddot{a}_{35:\overline{30}|} = \ddot{a}_{35} - v^{20}\frac{l_{65}}{l_{35}}\ddot{a}_{65} = 14.01456$ and $A_{35} = 0.1287194$

   so $P = \frac{12871.94}{0.95\times 14.01456 - 0.15} = \boxed{977.83}$

   (b) Calculate the reduction that Company A would apply to the sum insured.

   need $(100{,}000 - X)A_{39} = 977.83(0.95\,\ddot{a}_{39:\overline{30}|} - 0.15)$

   where $\ddot{a}_{39:\overline{30}|} = \ddot{a}_{39} - v^{20}\frac{l_{69}}{l_{39}}\ddot{a}_{69} = 13.81705$ and $A_{39} = 0.1542484$

   so $X = \boxed{17740.09}$

   (c) Calculate the gross premium the impaired life would pay under Company B's policy. (You will need some values that are not on the ILT - ask and we'll give them to you.)

   to incorporate the increased mortality, use $i^* = (1.06)e^{0.00939} - 1 = 7\%$

   at 7%, $\ddot{a}^*_{35} = 13.76895$ $A^*_{35} = 0.220626$ $\ddot{a}^*_{65} = 9.27889$

   so $\ddot{a}^*_{35:\overline{30}|} = \ddot{a}^*_{35} - (v^*)^{20}\frac{l_{65}}{l_{35}}\ddot{a}^*_{65} = 12.79413$

   and $P^* = \frac{100{,}000\,A^*_{35}}{0.95\,\ddot{a}^*_{35:\overline{30}|} - 0.15} = \boxed{1837.87}$

2. Think Like an Actuary

   (This is a true story) A few years ago my friend was trying to buy term life insurance with a sum insured of \$400,000. She wanted the beneficiary to be a charity, since she has no family and has dedicated much of her time and effort to charitable work. She was in excellent health, and a non-smoker aged 30. The insurance company offerred her a policy of \$100,000 if she paid "high-risk" premiums. Why would the insurer consider this to be a high risk policy? If you were the insurer, what information would you want if you were going to issue this policy?

   There may not exist a strong enough insurable interest if the charity would not be harmed by the death of the policyholder. The insurer would want to know the relationship (e.g. is she on the charity's board?) Also, this is unusual behaviour so the insurer may want to use more careful underwriting.

3. Calculate the standard deviation of L_0 for a 3-year fully discrete endowment insurance issued to (50), using the ILT at 6% interest. Assume: equivalence principle premium, no expenses.

   First, find $P = \frac{A_{50:\overline{3}|}}{\ddot{a}_{50:\overline{3}|}}$ $\quad \ddot{a}_{50:\overline{3}|} = 1 + \frac{l_{51}}{l_{50}}v + \frac{l_{52}}{l_{50}}v^2 = 2.816857$

   $A_{50:\overline{3}|} = 1 - d\,\ddot{a}_{50:\overline{3}|}$ so $P = 0.298402$

   $$L_0 = \begin{cases} v - P = 0.644994 & \text{with prob. } d_{50}/l_{50} \\ v^2 - P(1+v) = 0.310084 & \text{with prob. } d_{51}/l_{50} \\ v^3 - P(1+v+v^2) = -0.005870 & \text{with prob. } l_{52}/l_{50} \end{cases}$$

   $SD(L_0) = \sqrt{E[L_0^2] - E[L_0]^2}$, $E[L_0] = 0$

   $= \sqrt{0.00311} = \boxed{0.05577}$

   or $SD(L_0) = \left(1 + \frac{P}{d}\right)\sqrt{{}^2A_{50:\overline{3}|} - A_{50:\overline{3}|}^2}$ gives a similar result (difference in rounding)