The barbeque pool heater: An algorithm to construct tubular networks that occupy arbitrary regions in $\mathbb{R}^3$

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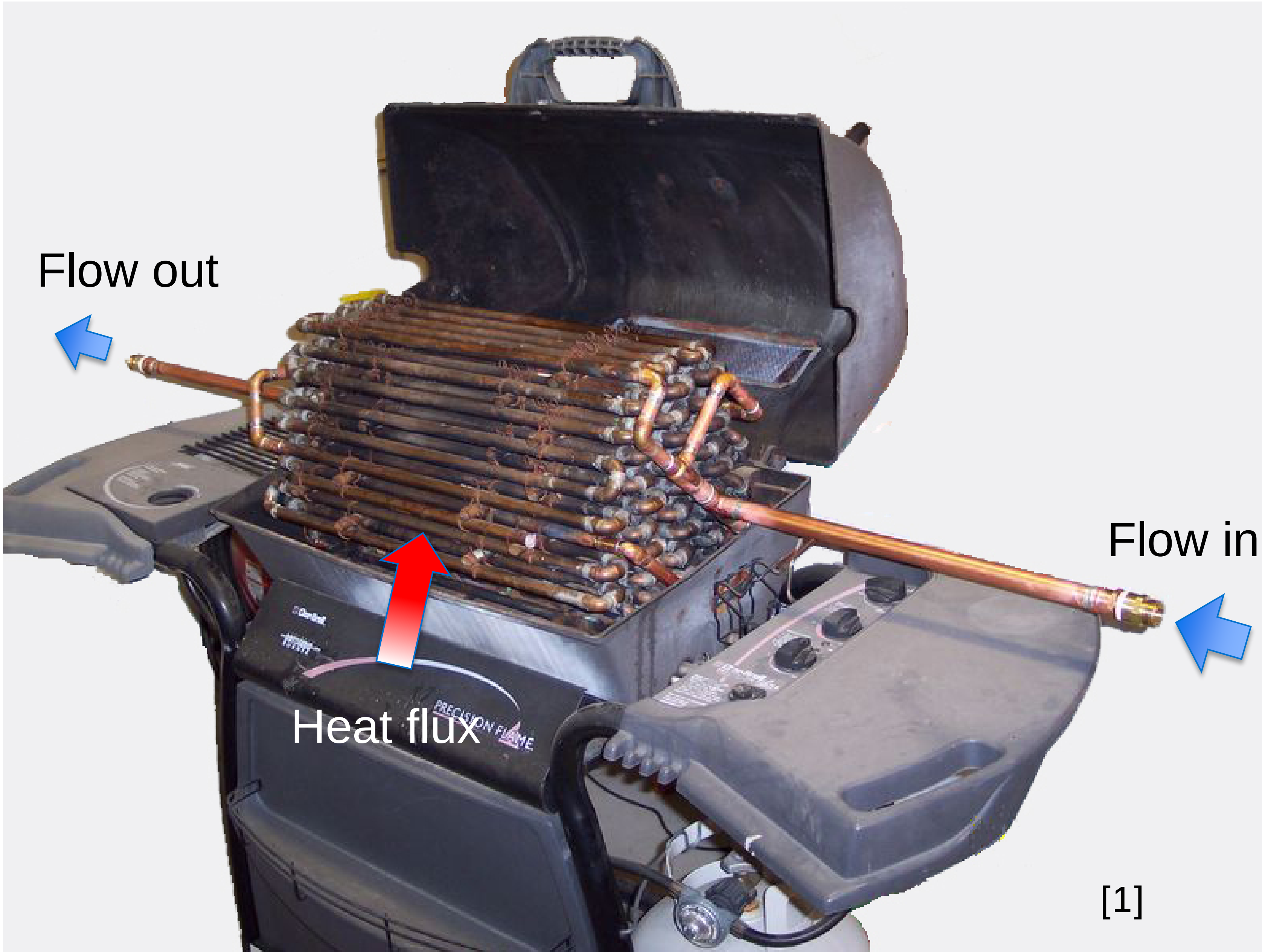
The barbeque pool heater

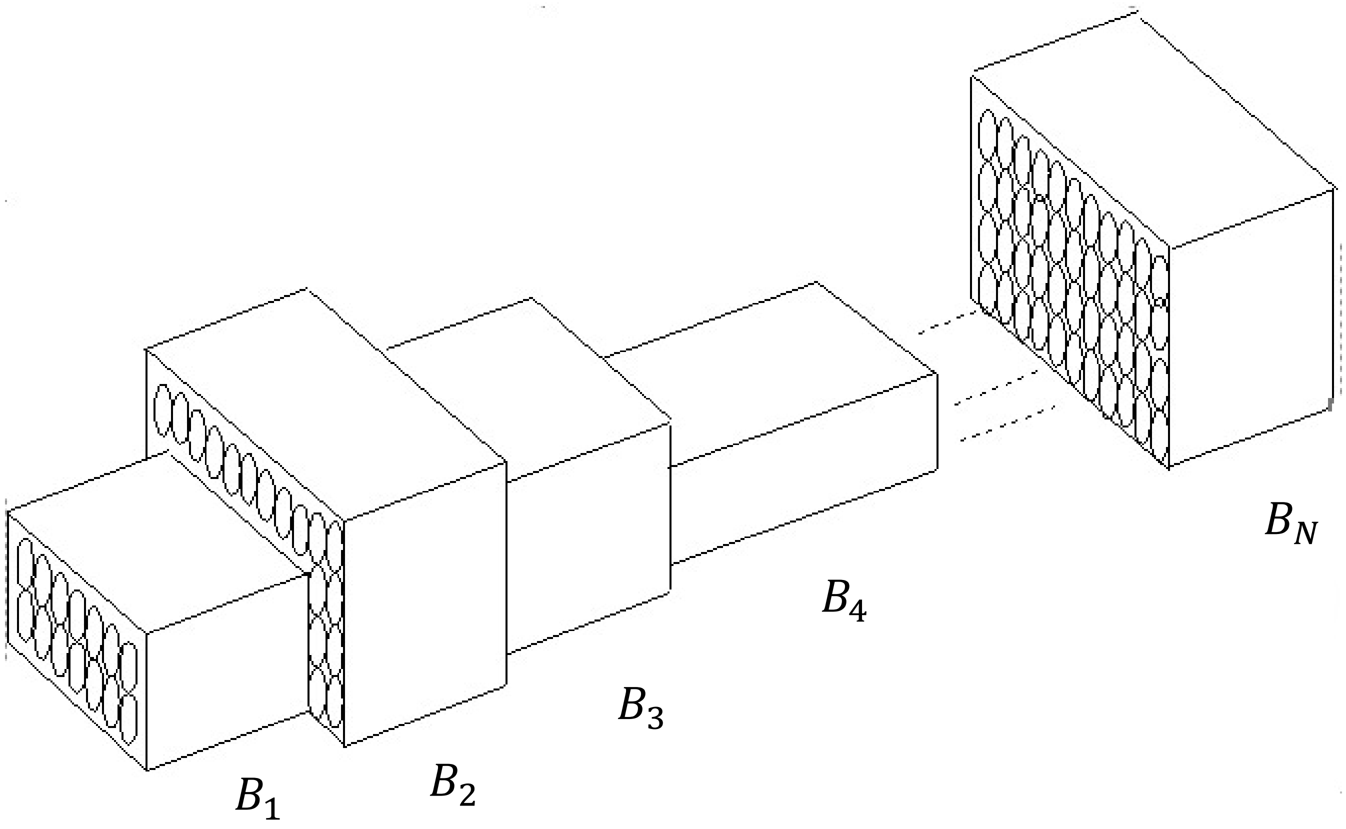


The General Approach

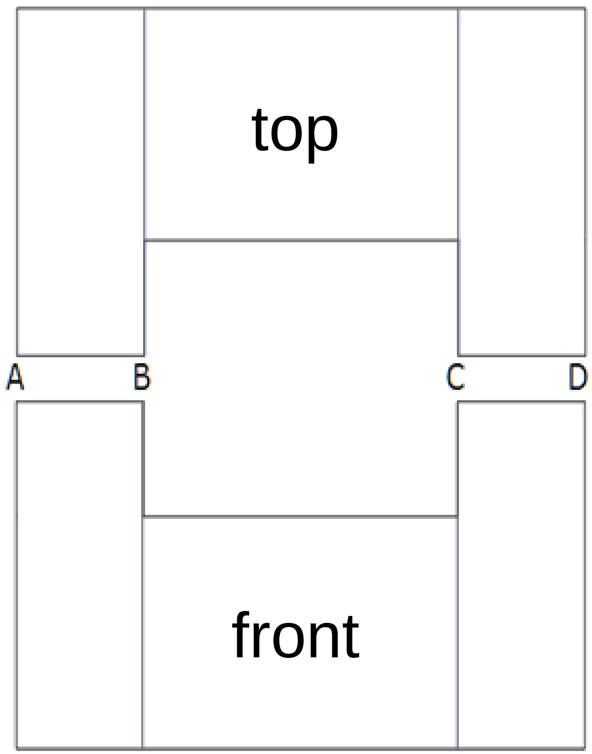
- Given a volume, discretize and pack sections with circles
- Connect the network with endcaps and internal connections
- Confirm the feasibility of the networks produced on topological grounds
- Then to determine the optimal pipe network for heat exchange:
  - Determine fluid flow subject to pressure boundary conditions
  - Given the flow distribution and appropriate thermal boundary conditions, estimate heat transfer characteristics of the pipe network

Spatial Discretization

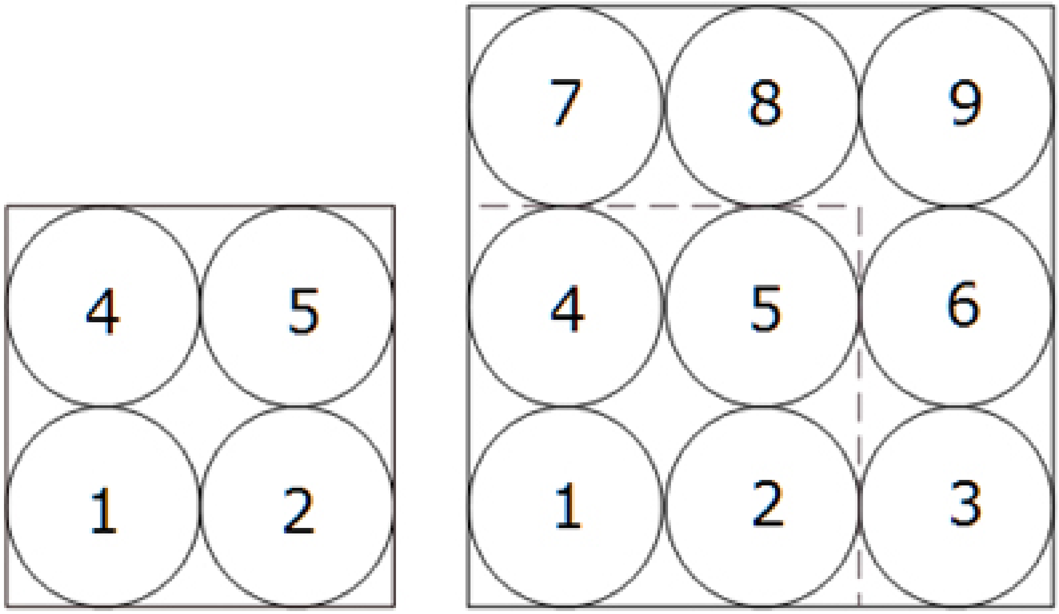
- Given an envelope, discretize into blocks B_i to obtain cross sections
- Pack different cross sections with circles to represent tubes
- Algorithmically, connect the tubes based on the packing



A Particular Envelope



Top and front views of "saddle bag"



section BC

sections AB & CD

Sectional views at A, B, C, and D revealing circle packings

Construction of Tubular Networks

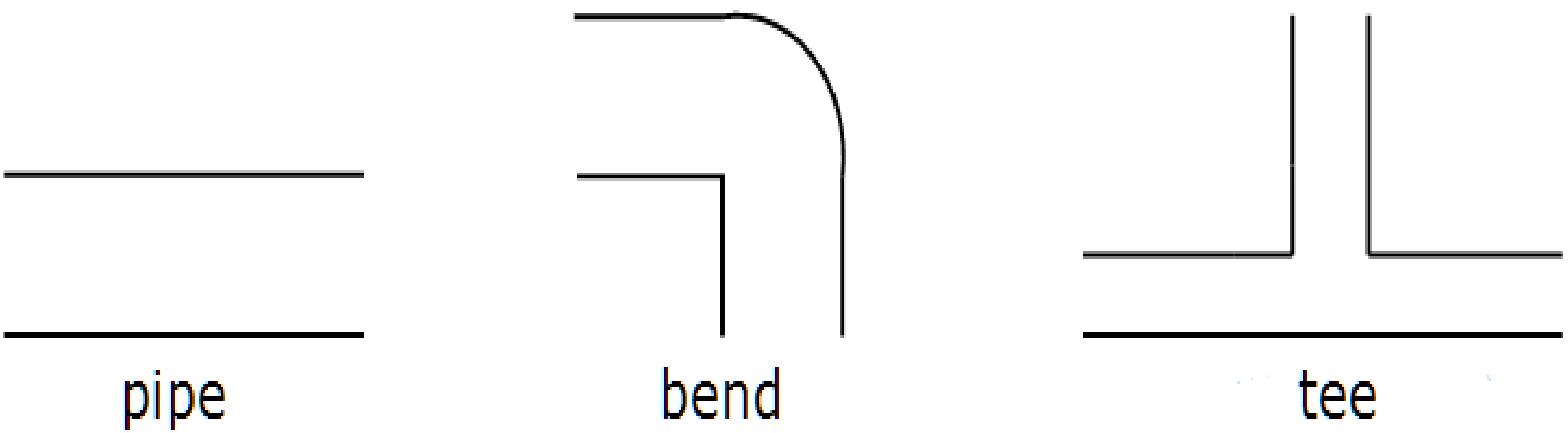
The connection algorithm:

- Step 1: Obtain a solution for each side to get a potential solution for the network connection problem.

- Step 2: Check feasibility of potential solutions.

Allowable Elements

Allow only these basic elements:

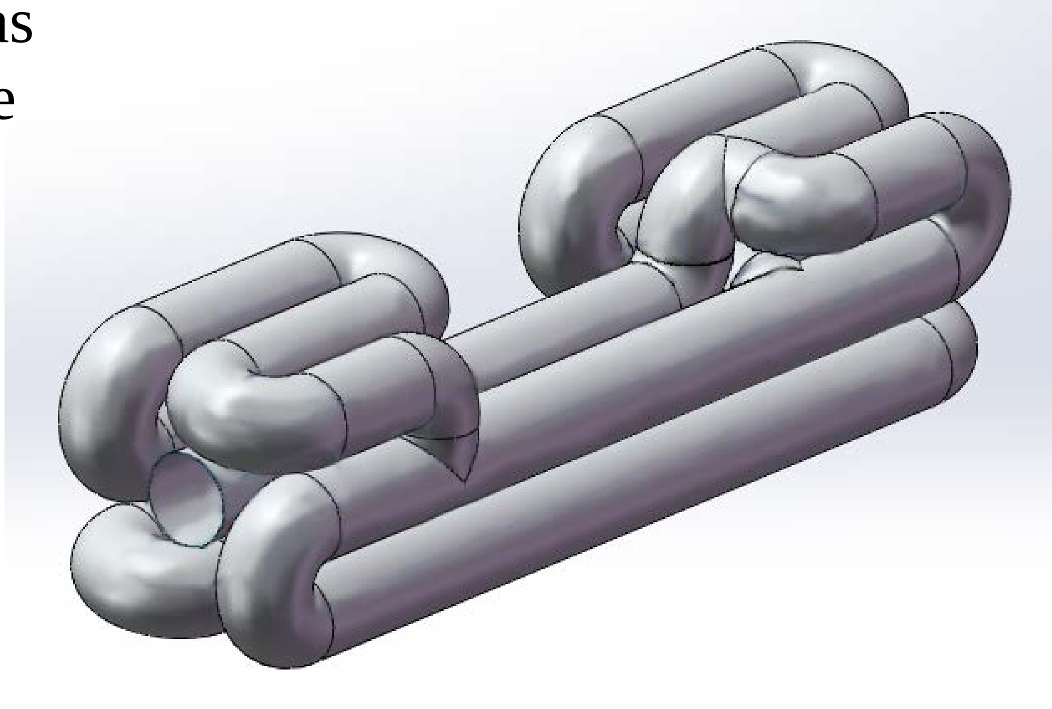


Step 1: Obtain a Feasible Solution for Each Side

For outer ends:

- Use only end caps to connect tubes and leave two tubes as inlet and outlet of the entire network.

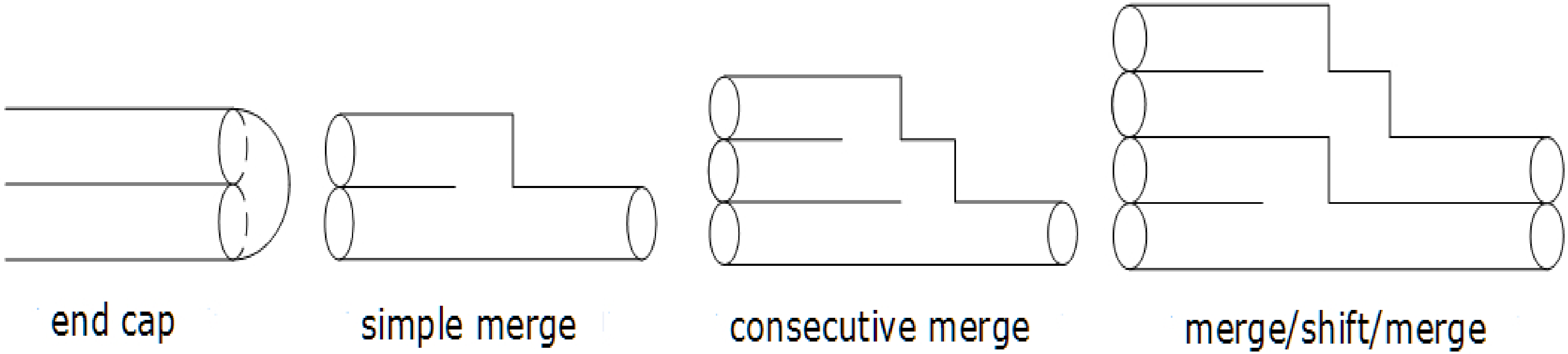
- Use Minimum Degree Matching Algorithm and Depth-first Search to enumerate all possible solutions for one end.



Step 1: Obtain a Feasible Solution for Each Side

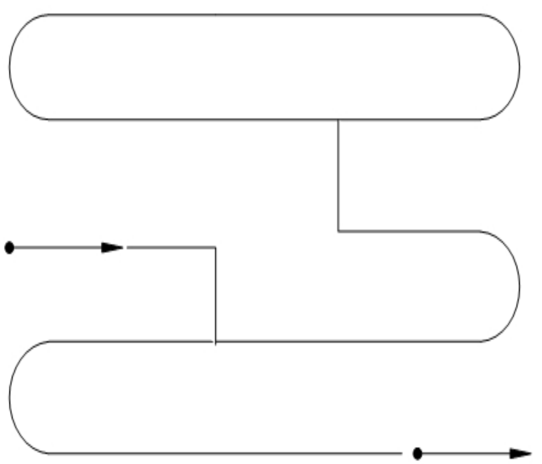
For middle sides:

- Employ the operations in the figure below
- Use Unified Minimum Degree Matching Algorithm combined with Depth-first Search to enumerate all possible solutions for one middle side

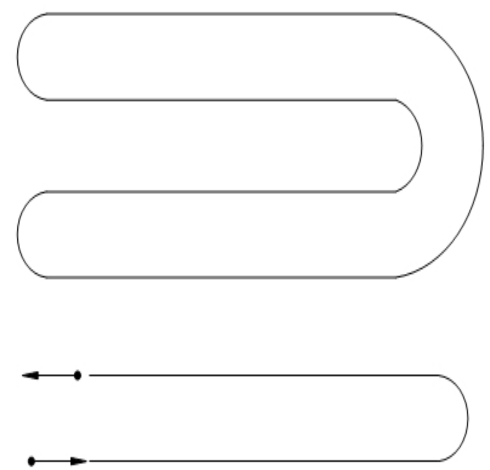


Step 2: Verify Feasibility

- Construct a graph from the potential solution.
- Determine whether dead ends exist (bridges)
- Determine whether the graph is connected.



dead end



disconnected graph

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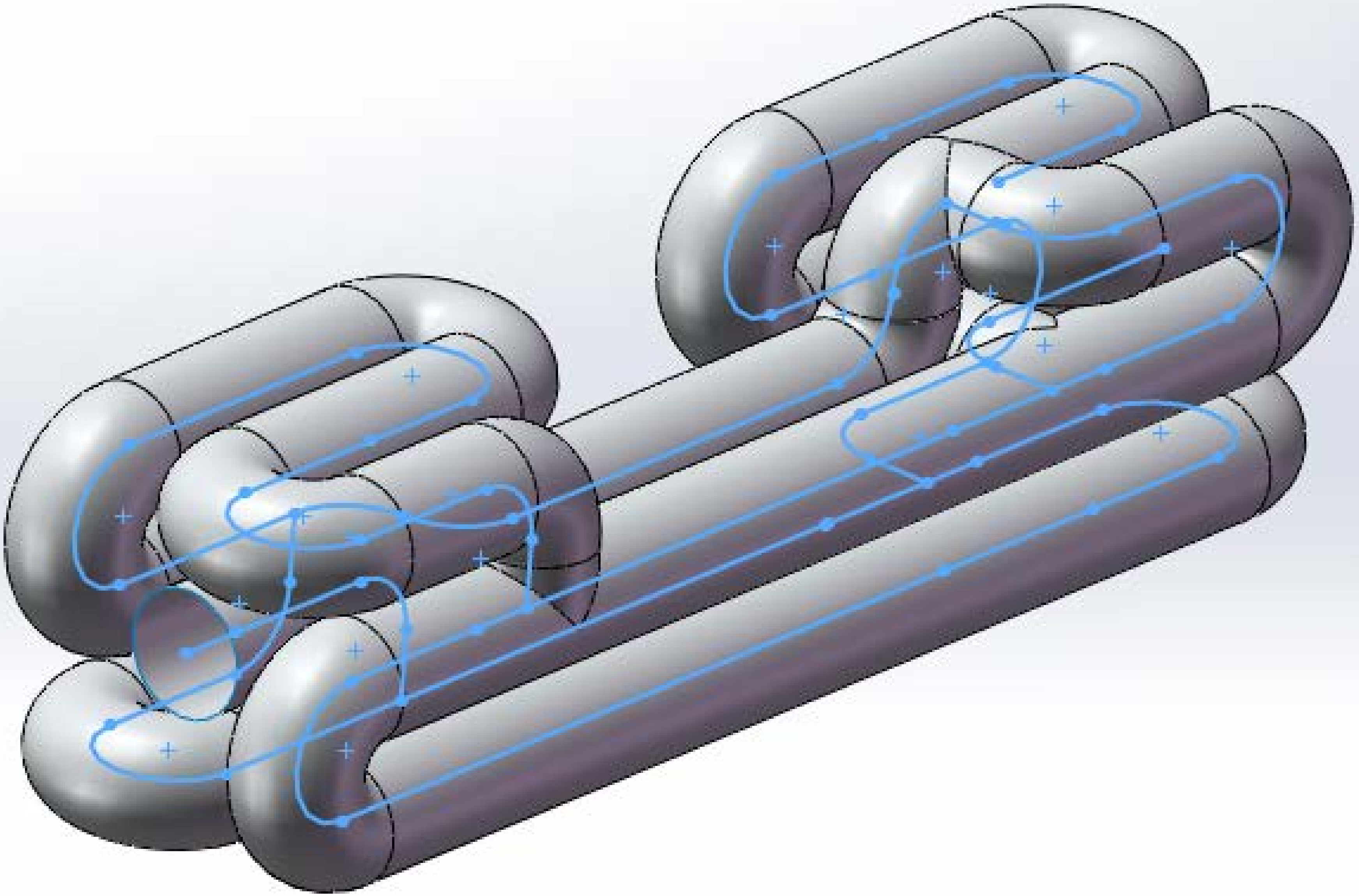
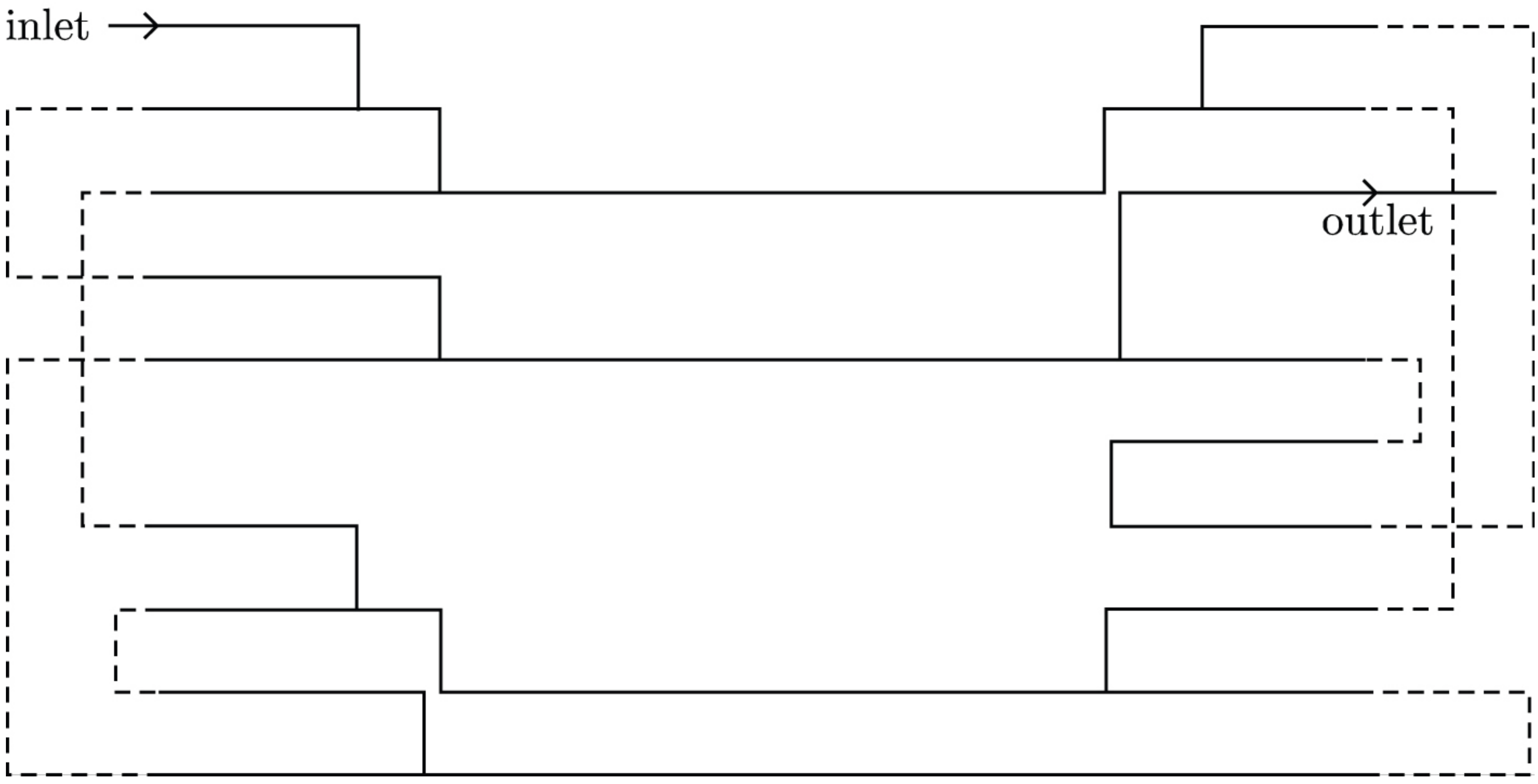
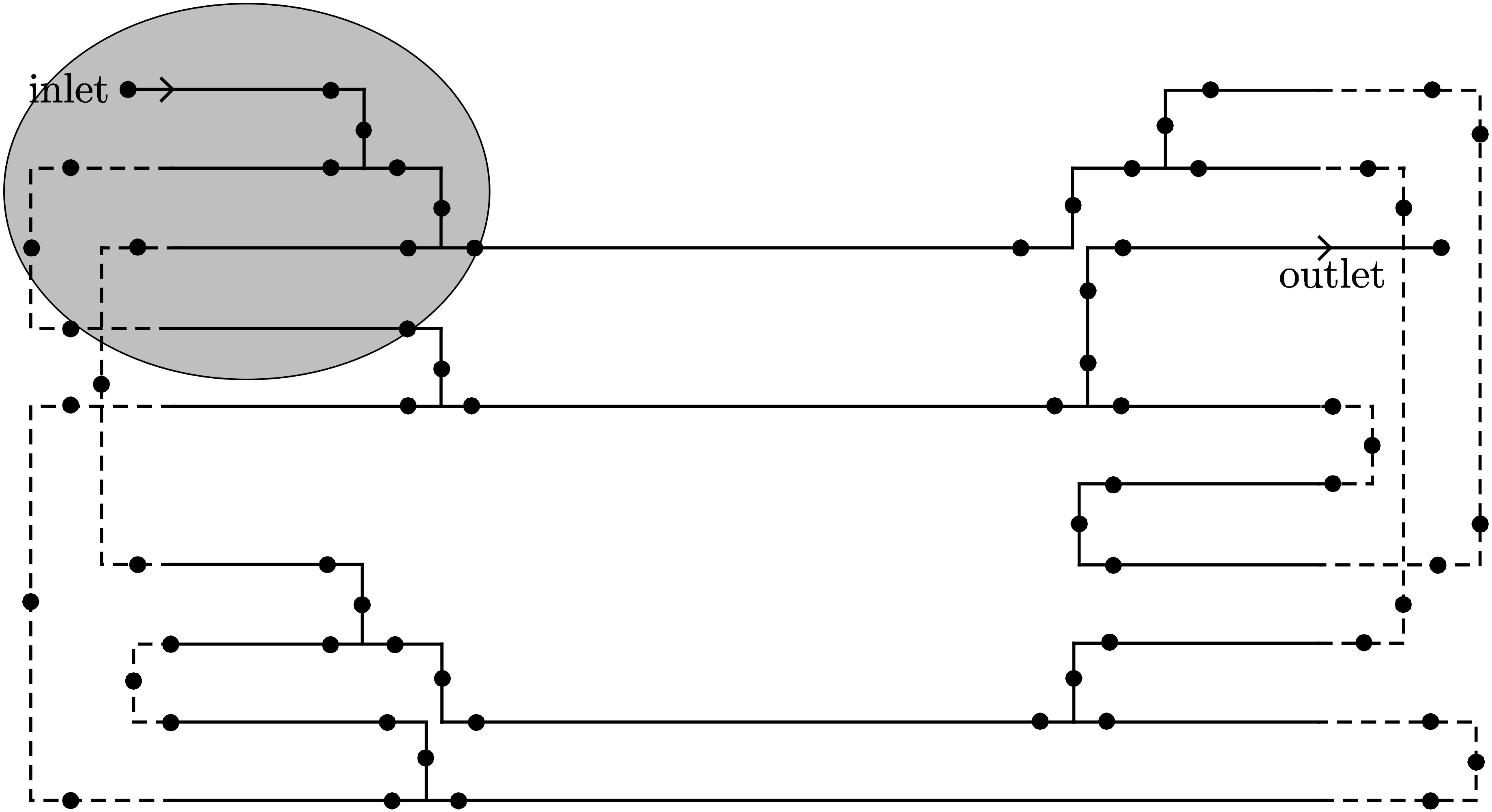
Given One of 6798 Feasible Solutions



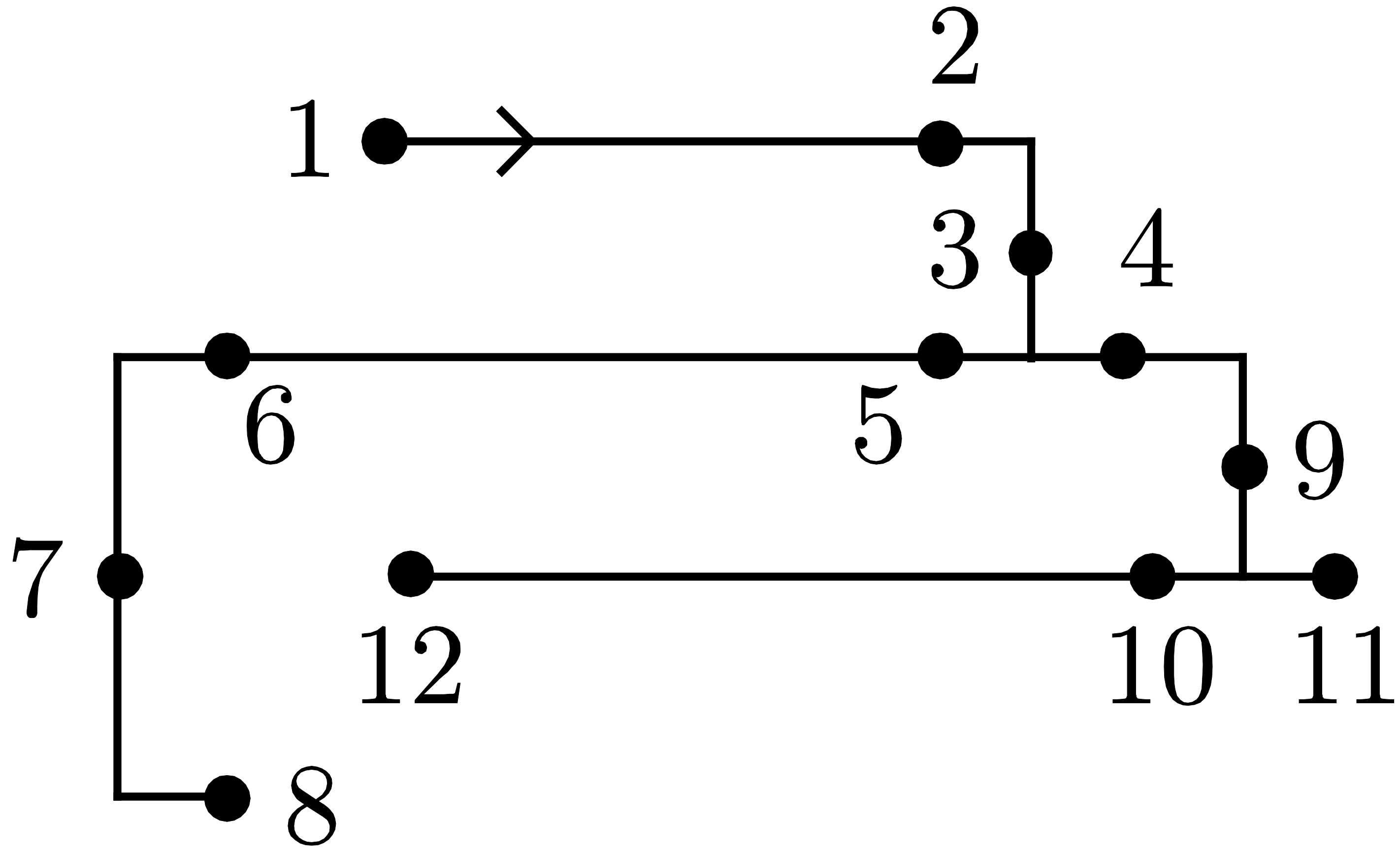
Abstract Network Representation



Element-wise Network Decomposition

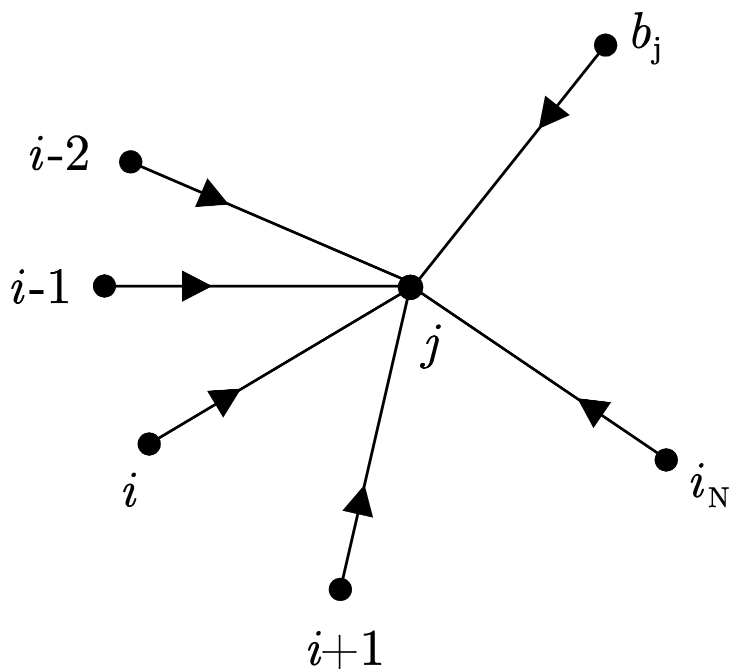


Partial Network Near Inlet



Conservation Equations:

Conservation of mass

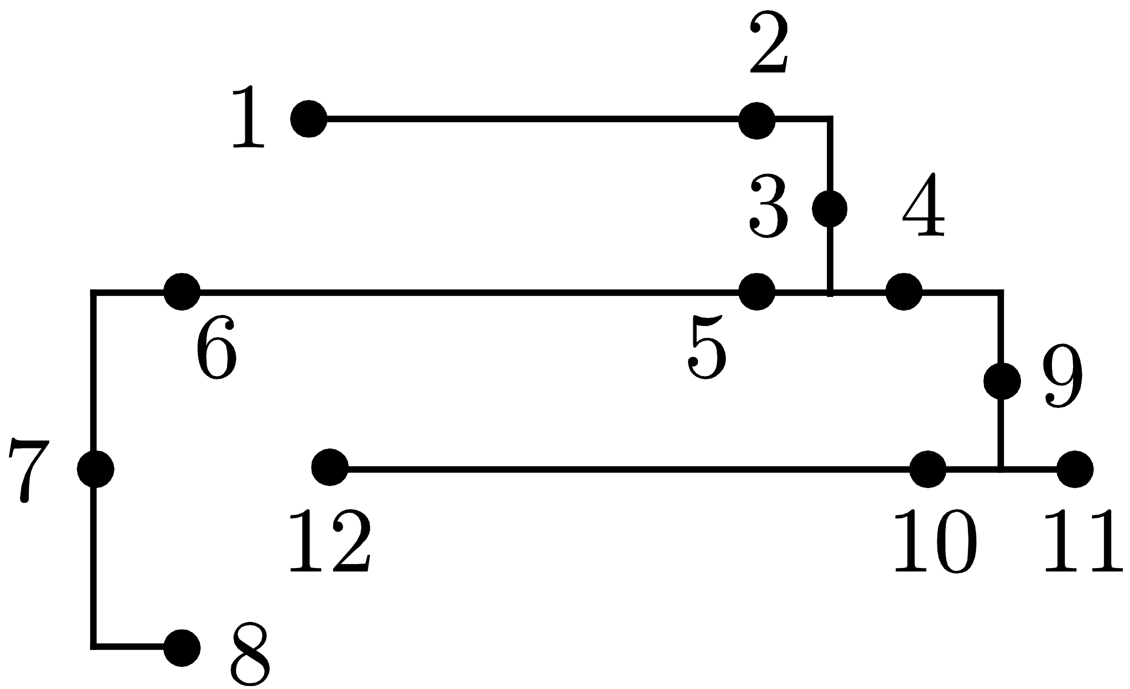


$$\sum_{i=1}^{N} \dot{m}_{ij} + b_j = 0$$

$$(\dot{m}_{ij} = -\dot{m}_{ji})$$

$\dot{m}_{ij}$ is the mass flow rate [kg/s] from node i to node j and b_j is either a source or a sink of mass, (positive at inlets, negative at outlets, and zero at internal nodes)

Conservation of Mass



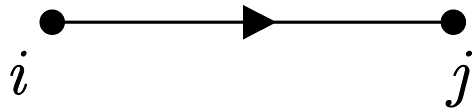
$$\dot{m}_{21} + b_1 = 0$$

$$\dot{m}_{12} + \dot{m}_{32} = 0$$

$$\vdots$$

Conservation Equations:

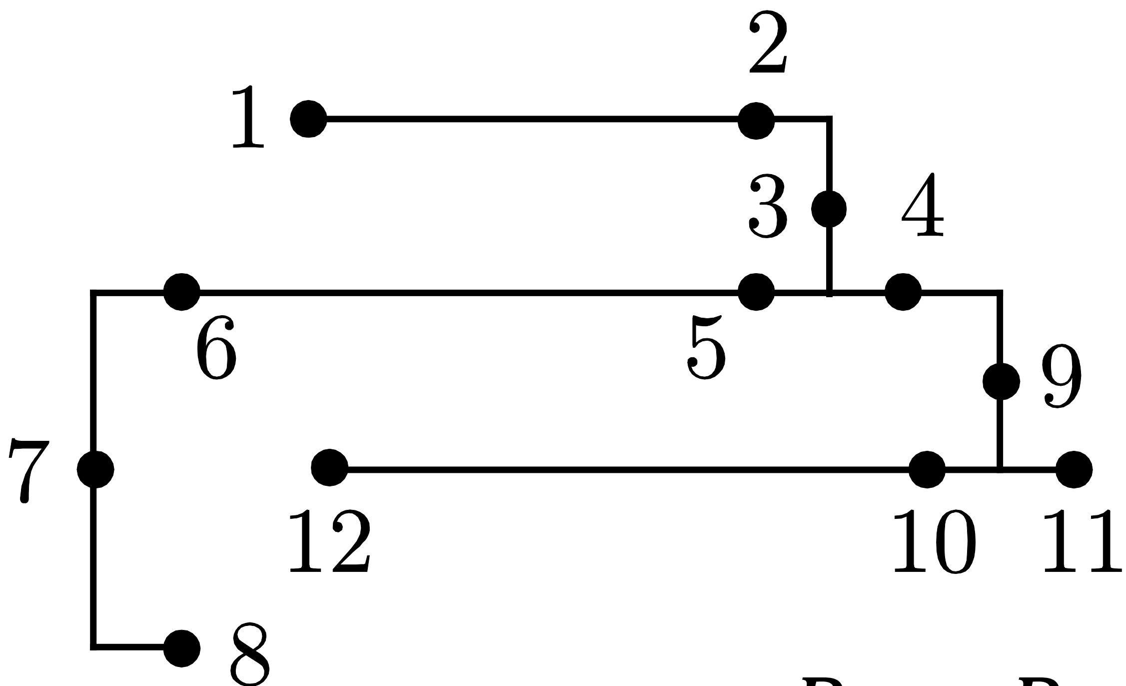
Conservation of energy



$$P_i - P_j = f_{ij}(\dot{m}_{ij}, \text{geometry})$$

where P_i is the pressure [Pa] at node *i* and f_{ij} is the pressure drop in branch *ij* as a function of the flow through and the geometry of the branch

Conservation of Energy



$$P_1 = P_{\text{inlet}}$$

$$P_1 - P_2 = f_{1,2}(\dot{m}_{12}, \text{geometry})$$

$$P_2 - P_3 = f_{2,3}(\dot{m}_{23}, \text{geometry})$$

$$\vdots$$

A System of Linear Equations:

$$\dot{m}_{21} + b_1 = 0$$

$$\dot{m}_{12} + \dot{m}_{32} = 0$$

$$\vdots$$

$$P_1 = P_{\text{inlet}}$$

$$P_1 - P_2 = f_{1,2}(\dot{m}_{12}, \text{geometry})$$

$$P_2 - P_3 = f_{2,3}(\dot{m}_{23}, \text{geometry})$$

$$\vdots$$

$$\begin{pmatrix} -1 & 0 & \dots & 0 & 1 & 0 & & & \dots & \\ 1 & -1 & \dots & & 0 & & & & \dots & \\ & & & & \vdots & & & & & \\ & & & & & \ddots & & & & \\ & & & & & & 1 & & & \\ & & & & & & 1 & -1 & & \\ & & & & & & & 1 & -1 & \\ & & & & & & & & & \ddots \end{pmatrix} \begin{pmatrix} \dot{m}_{12} \\ \vdots \\ \dot{m}_{N-1,N} \\ b_1 \\ \vdots \\ b_N \\ P_1 \\ P_1 \\ P_2 \\ \vdots \\ P_{\mathrm{N}} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ P_{\text{inlet}} \\ f_{12} \\ f_{23} \\ \vdots \\ P_{\text{outlet}} \end{pmatrix}$$

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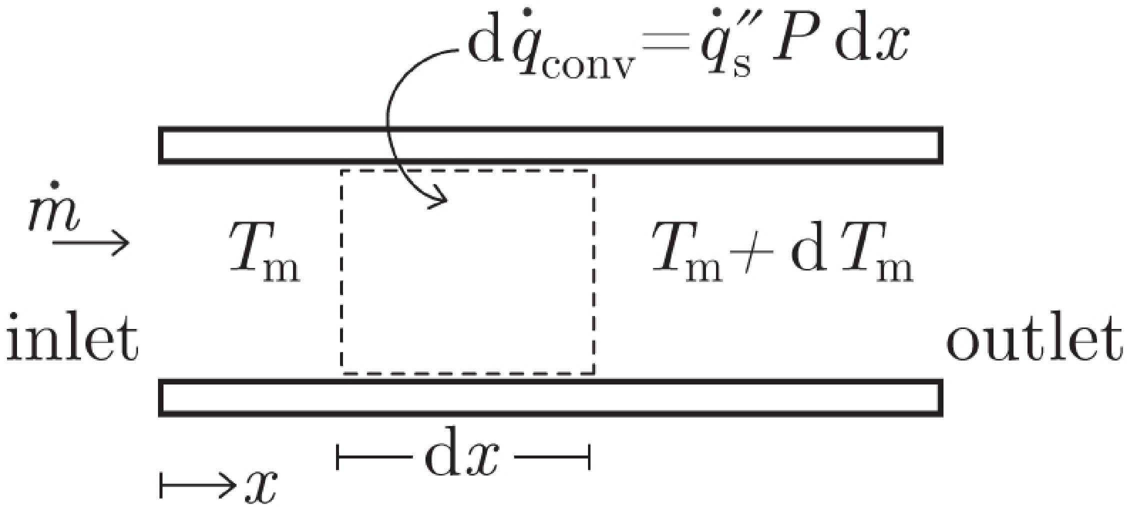
The Heat Transfer Problem:



$$\mathrm{d}\dot{q}_{\mathrm{conv}} = \dot{m}c_p\big((T_{\mathrm{m}} + \mathrm{d}T_{\mathrm{m}}) - T_{\mathrm{m}}\big) = \dot{m}c_p \mathrm{d}T_{\mathrm{m}}$$

but $\mathrm{d}\dot{q}_{\mathrm{conv}} = \dot{q}_s{}'' P \, \mathrm{d}x$ and $\dot{q}_s{}'' = h(T_{\mathrm{s}} - T_{\mathrm{m}})$

so $$\frac{\mathrm{d}T_{\mathrm{m}}}{\mathrm{d}x} = \frac{P}{\dot{m}c_p} h(T_{\mathrm{s}} - T_{\mathrm{m}})$$

The Heat Transfer Problem:

With $\Delta T = T_\mathrm{s} - T_\mathrm{m}$ and T_s constant, $\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x} = -\frac{\mathrm{d}\Delta T}{\mathrm{d}x}$,

so $\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x} = \frac{P}{\dot{m}c_p} h(T_\mathrm{s} - T_\mathrm{m}) \Rightarrow -\frac{\mathrm{d}\Delta T}{\mathrm{d}x} = \frac{P}{\dot{m}c_p} h\Delta T$

$$\int_{\Delta T_i}^{\Delta T_o} \frac{\mathrm{d}\Delta T}{\Delta T} = -\frac{PL}{\dot{m}c_p}\frac{1}{L}\int_0^L h\,\mathrm{d}x$$

and, with $\bar{h} = \frac{1}{L}\int_0^L h\,\mathrm{d}x$ and $A_\mathrm{s} = PL$,

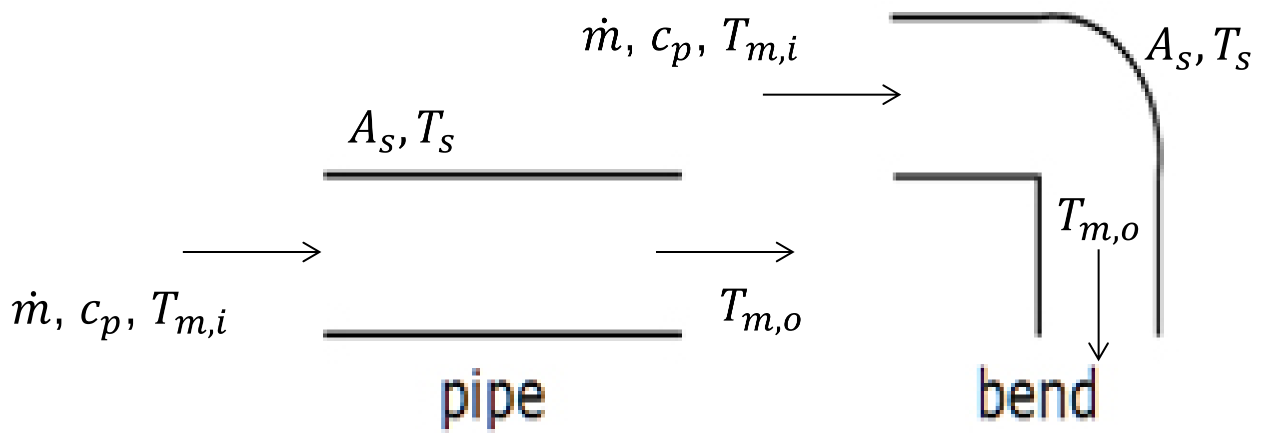
$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{A_\mathrm{s}}{\dot{m}c_p}\bar{h}$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\mathrm{s} - T_{\mathrm{m},o}}{T_\mathrm{s} - T_{\mathrm{m},i}} = e^{\left(-\frac{A_\mathrm{s}}{\dot{m}c_p}\bar{h}\right)}$$

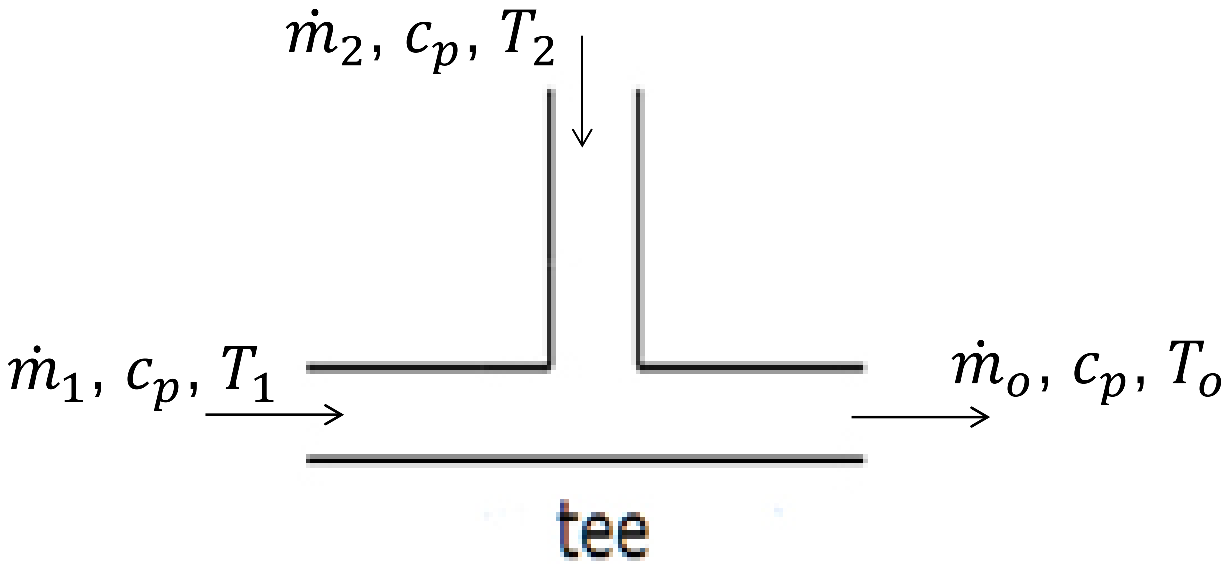
The Heat Transfer Problem:

$$\frac{T_{\mathrm{s}} - T_{\mathrm{m},o}}{T_{\mathrm{s}} - T_{\mathrm{m},i}} = e^{-\frac{A_s \bar{h}}{\dot{m} c_p}}$$

$$T_s \left(1 - e^{\frac{-A_s \bar{h}}{\dot{m} c_p}}\right) = T_{m,o} - T_{m,i}\, e^{\frac{-A_s \bar{h}}{\dot{m} c_p}}$$



The Heat Transfer Problem:



$$T_o = \sum_{inlet\ i} \frac{\dot{m}_i}{\sum_{outlet\ j} \dot{m}_j} T_i$$

Results:

- The overall heat transfer to the pool is given by

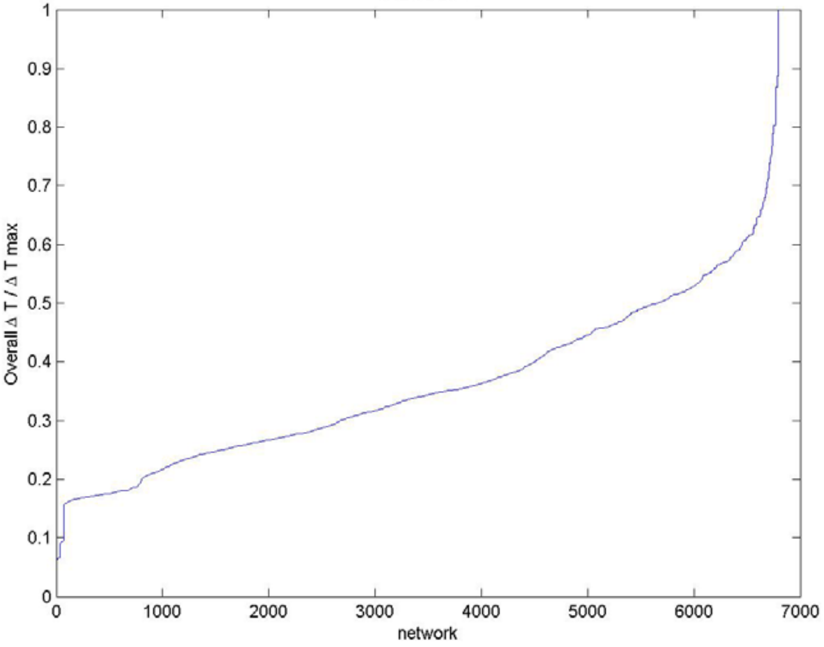
$$\dot{q}_o = \dot{m} c_p T_o$$

computed at the outlet of the heat exchanger.

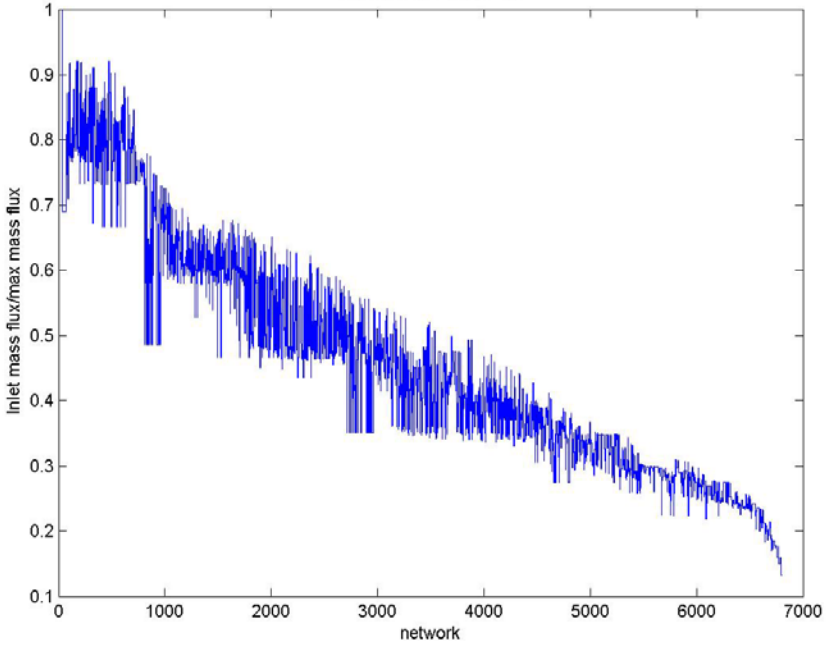
- For the sake of computational efficiency, are we able to infer anything about the overall heat transfer characteristics from the geometry or flow alone?

Results:

Ranked ΔT



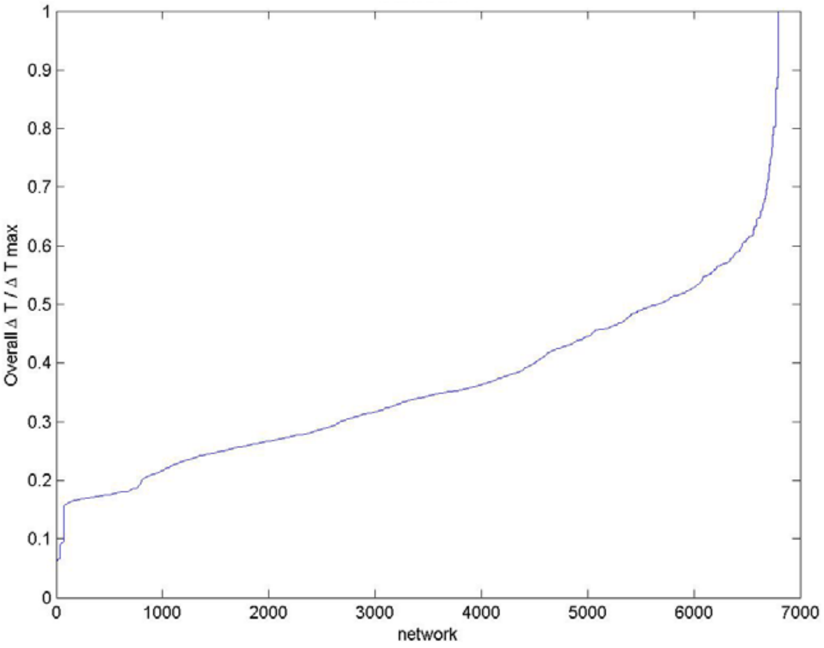
corresponding ranking of $\dot{m}$



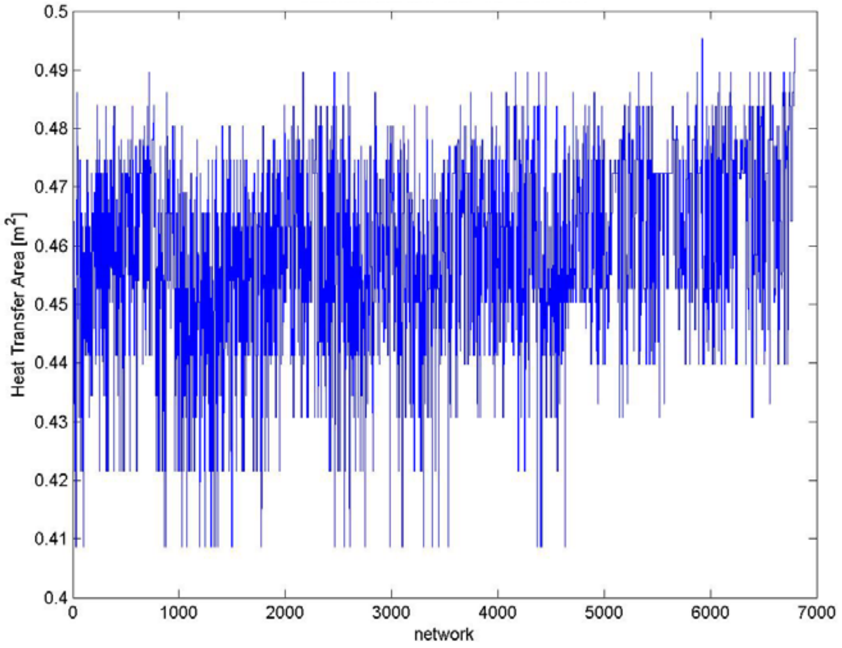
correlation of ΔT and inlet $\dot{m}$ is -0.8981

Results:

Ranked ΔT



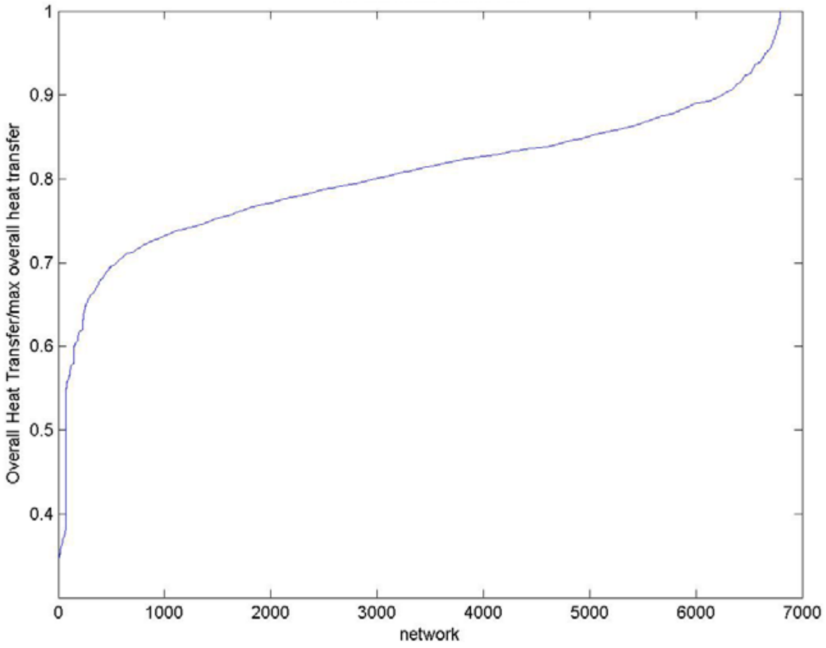
corresponding ranking of A_s



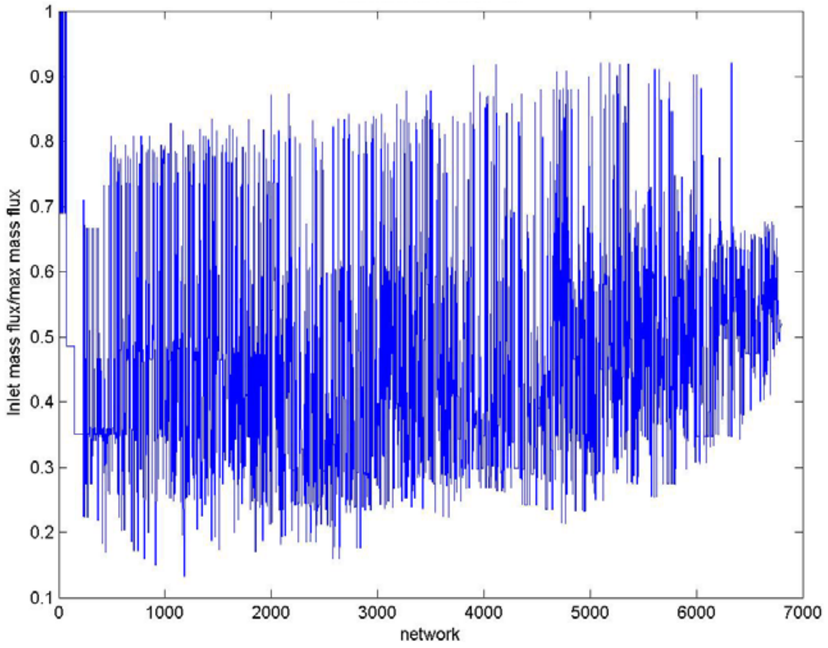
correlation of ΔT and A_s is 0.2854

Results:

Ranked $\dot{q}$



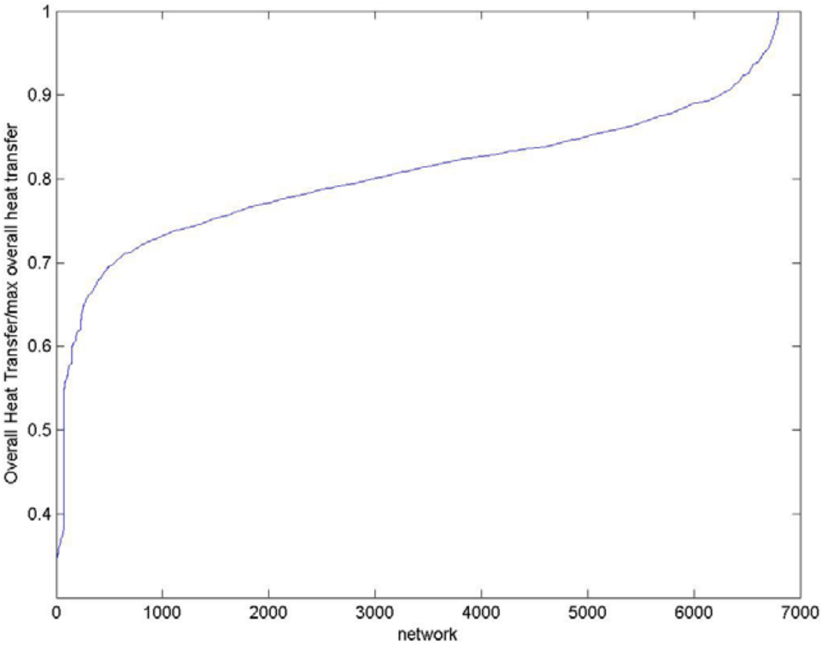
corresponding ranking of $\dot{m}$



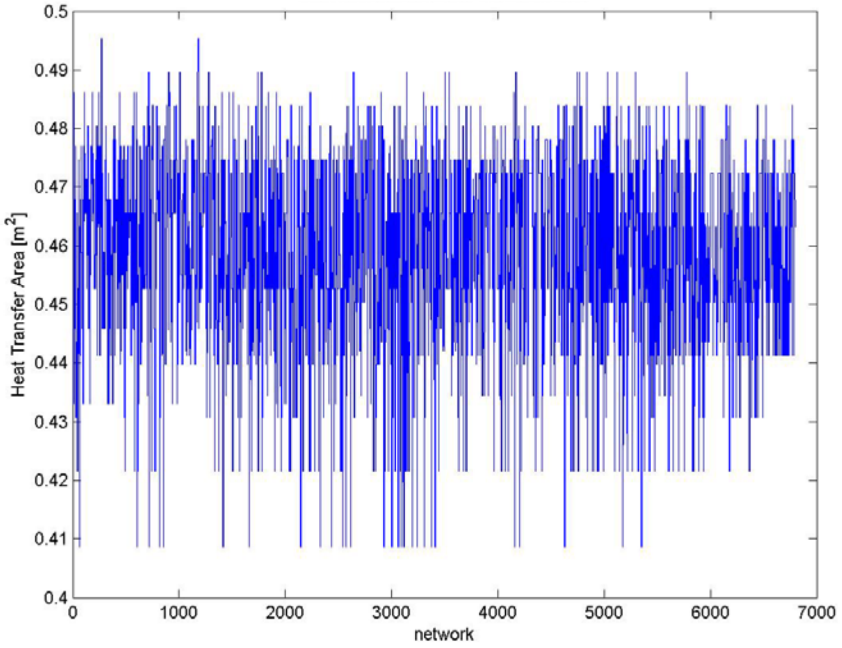
correlation of $\dot{q}$ and $\dot{m}$ is -4.1705e-04

Results:

Ranked $\dot{q}$



corresponding ranking of A_s



correlation of heat flux $\dot{q}$ and A_s is -0.0146

Conclusions

- For the given envelope, discretization, and circle packing, the network connection algorithm produces 6000+ feasible networks
- Neither outlet temperatures nor overall heat transfers exhibit much correlation with the overall heat transfer surface area
- Whereas outlet temperatures exhibit strong negative correlations with mass flow rates, overall heat transfer is uncorrelated with inlet mass flow
- The overall characterization of the heat exchanger cannot be determined from the geometry or from the fluid flow alone

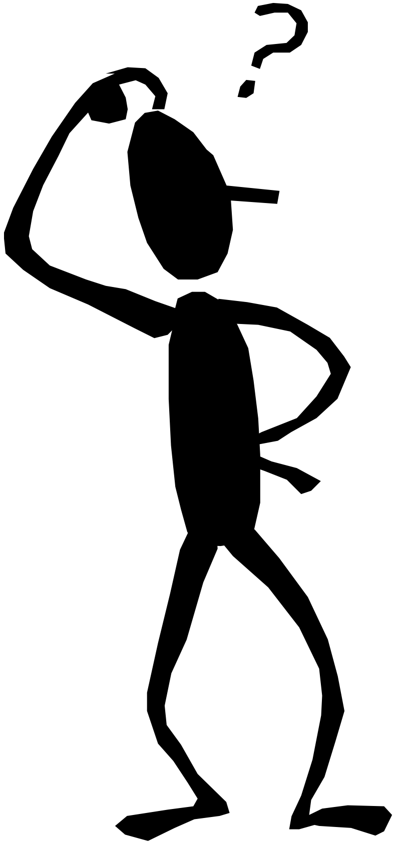
References

[1] http://www.redneckpoolheater.com/more/2006-11-26_Redneck_Pool_Heater_Manifold/images/100_1180.jpg.

[2] Rennels, Donald C. and Hudson, Hobart M., *Pipe Flow: A Practical and Comprehensive Guide*. John Wiley & Sons, Hoboken, New Jersey, 2012.

[3] Incropera, Frank P. and DeWitt, David P., *Fundamentals of Heat and Mass Transfer, 5 ed.*. John Wiley & Sons, Hoboken, New Jersey, 2001.

Questions



Thank You !