Examining the Role of Scale in the Context of the Non-Local-Means Filter

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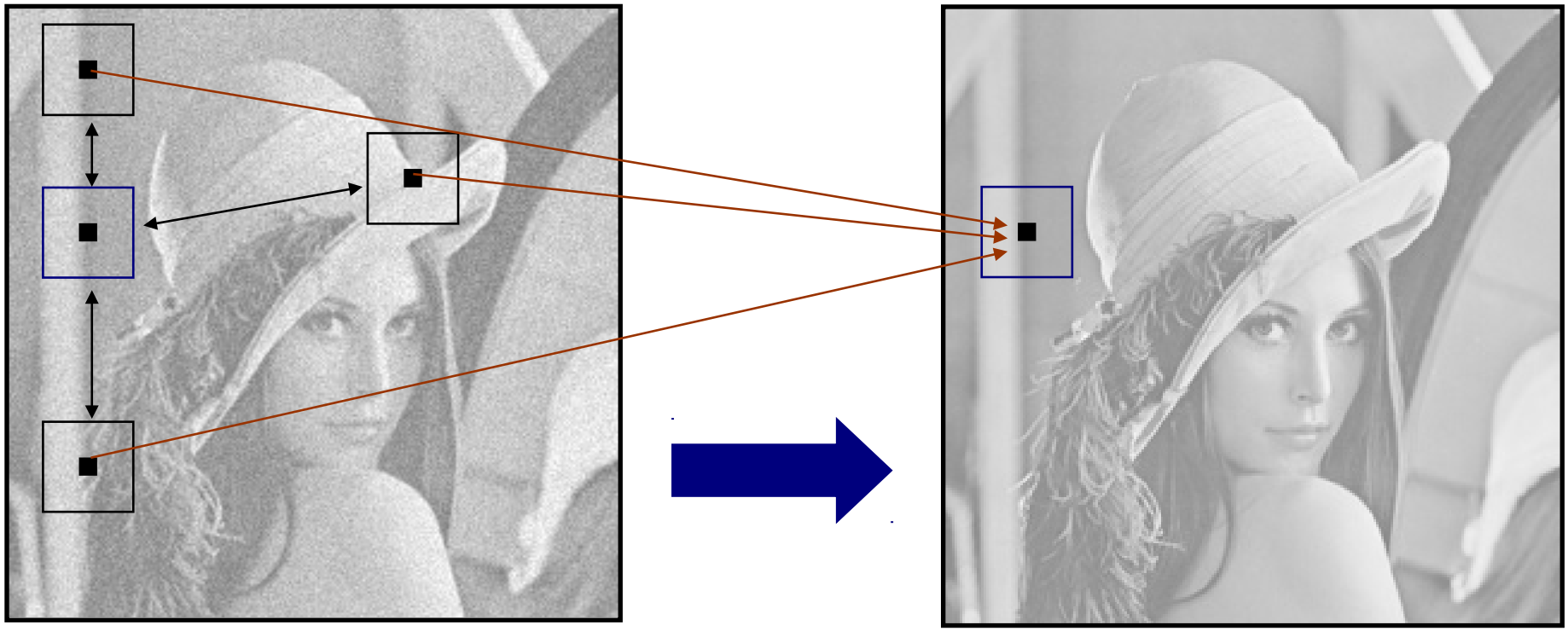
Agenda

- Nonlocal-means (NL-means) denoising
- An example-based variant of the NL-means
- Same-scale vs. cross-scale self-similarity
- Distribution of neighborhood distances at various scales
- Computational results on same-scale vs. cross-scale approximations
- Concluding remarks

Nonlocal-means (NL-means) denoising



(A. Buades, B. Coll, J. M. Morel, 2005)

The intensity of each pixel in the noisy image is replaced with the weighted average of all of the pixel intensities in the same image. The weights are determined based on neighborhood similarity of the pixels. Neighborhoods are typically assumed to be square patches of odd length.

Nonlocal-means (NL-means) denoising

(A. Buades, B. Coll, J. M. Morel, 2005)

$$\Omega = [1, \ldots, m] \times [1, \ldots, n]$$

$$\mathbf{u} = \mathbf{f} + \mathbf{n}. \qquad \mathbf{u}, \mathbf{f} \in l^2(\Omega)$$

For any $x \in \Omega$ define the denoising formula

$$NL\big(\mathbf{u}(x)\big) = \widetilde{\mathbf{f}}(x) = \frac{1}{C(x)} \sum_{y \in \Omega} w(x,y)\mathbf{u}(y), \text{ such that}$$

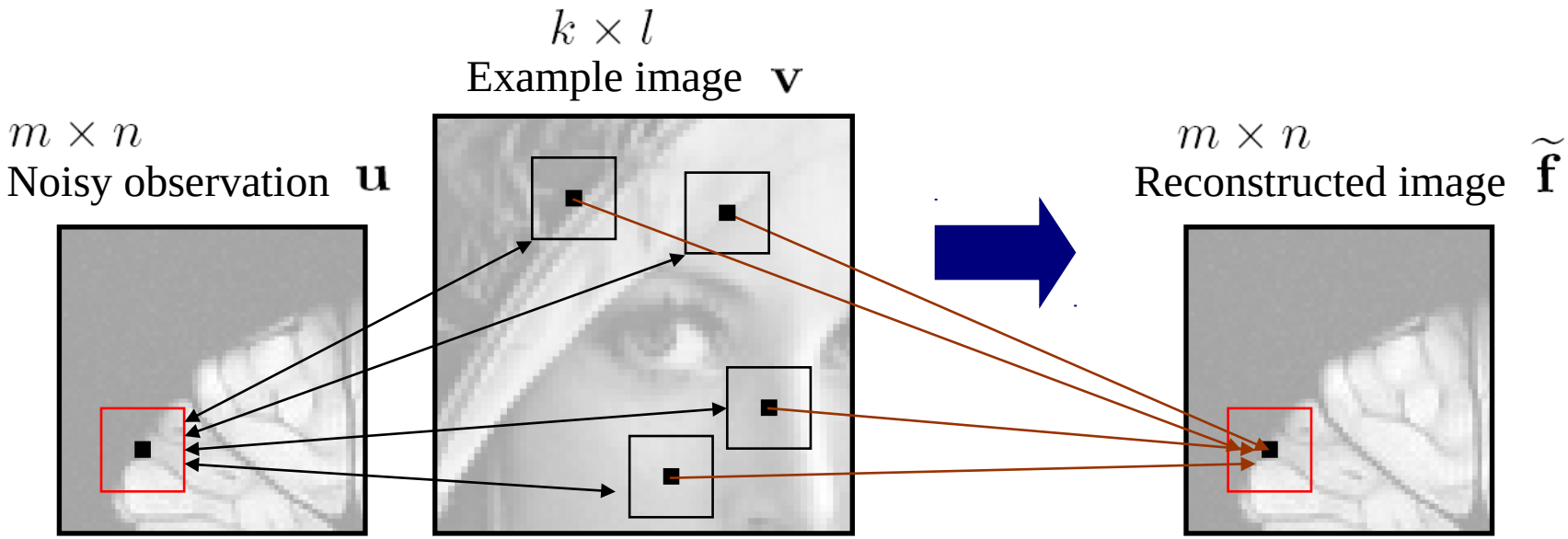
$$w(x,y) = \exp\Big(- \frac{\| \mathbf{u}(\mathcal{N}^d\{x\}) - \mathbf{u}(\mathcal{N}^d\{y\}) \|_{2,a}^2}{h^2} \Big), \text{ and}$$

$$C(x) = \sum_{y \in \Omega} w(x,y)$$

<u>Neighborhoods:</u> For any point in the domain of observation $(i,j) \in \Omega$, define

$$\mathcal{N}^d\{(i,j)\} = \Big\{ (i + i_1, j + j_1) \mid (i_1, j_1) \in \mathbb{Z}^2, \max\{|i_1|, |j_1|\} \leq d \Big\}.$$

An example-based variant of the NL-means



1-Weights are determined based on the similarity of the patches.

2-Weighted averages of central pixels are replaced in the corresponding location.

An example-based variant of the NL-means

(A natural extension of the NL-means denoising)

Example image: The notion of *example image* is denoted by $\mathbf{v} \in l^2(\Phi)$, where Φ is the $k \times l$ pixel lattice defined by

$$\Phi = [1, \ldots, k] \times [1, \ldots, l].$$

Algorithm 1: An Example-Based Variant of the NL-Means Filter. The approximation of $\mathbf{f}$, denoted by $\widetilde{\mathbf{f}}$, given the observation $\mathbf{u}$, using *example image* $\mathbf{v}$, is computed in the following way. For any $x \in \Omega$,

$$\widetilde{\mathbf{f}}_{(\mathbf{u},\mathbf{v},h,d,a)}(x) = \frac{1}{C(x)} \sum_{y \in \Phi} w(x,y)\mathbf{v}(y),$$

where

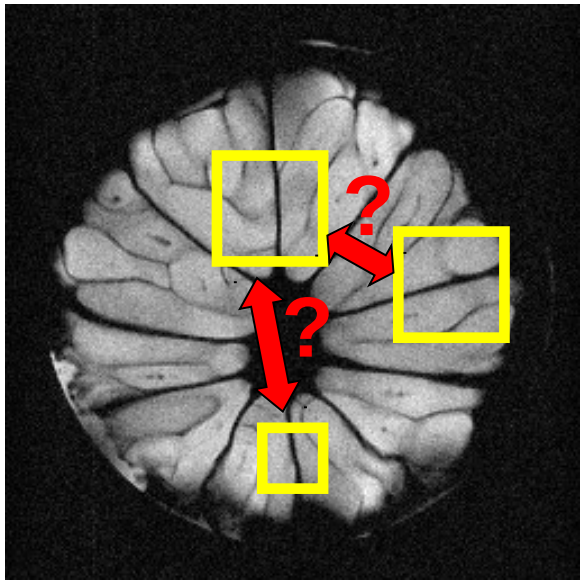
$$w(x,y) = \exp\left(- \frac{\| \mathbf{u}(\mathcal{N}^d\{x\}) - \mathbf{v}(\mathcal{N}^d\{y\}) \|_{2,a}^2}{h^2} \right)$$

and

$$C(x) = \sum_{y \in \Phi} w(x,y).$$

Same-scale vs. Cross-scale self-similarity

Question. What examples to choose?



There is a great deal of **cross-scale self-similarity** in natural images.

(PhD thesis, S. Alexander, Dept. of Applied Mathematics, University of Waterloo, 2005)

Same-scale vs. Cross-scale self-similarity

In ICIAR 2007, we presented an image zooming technique

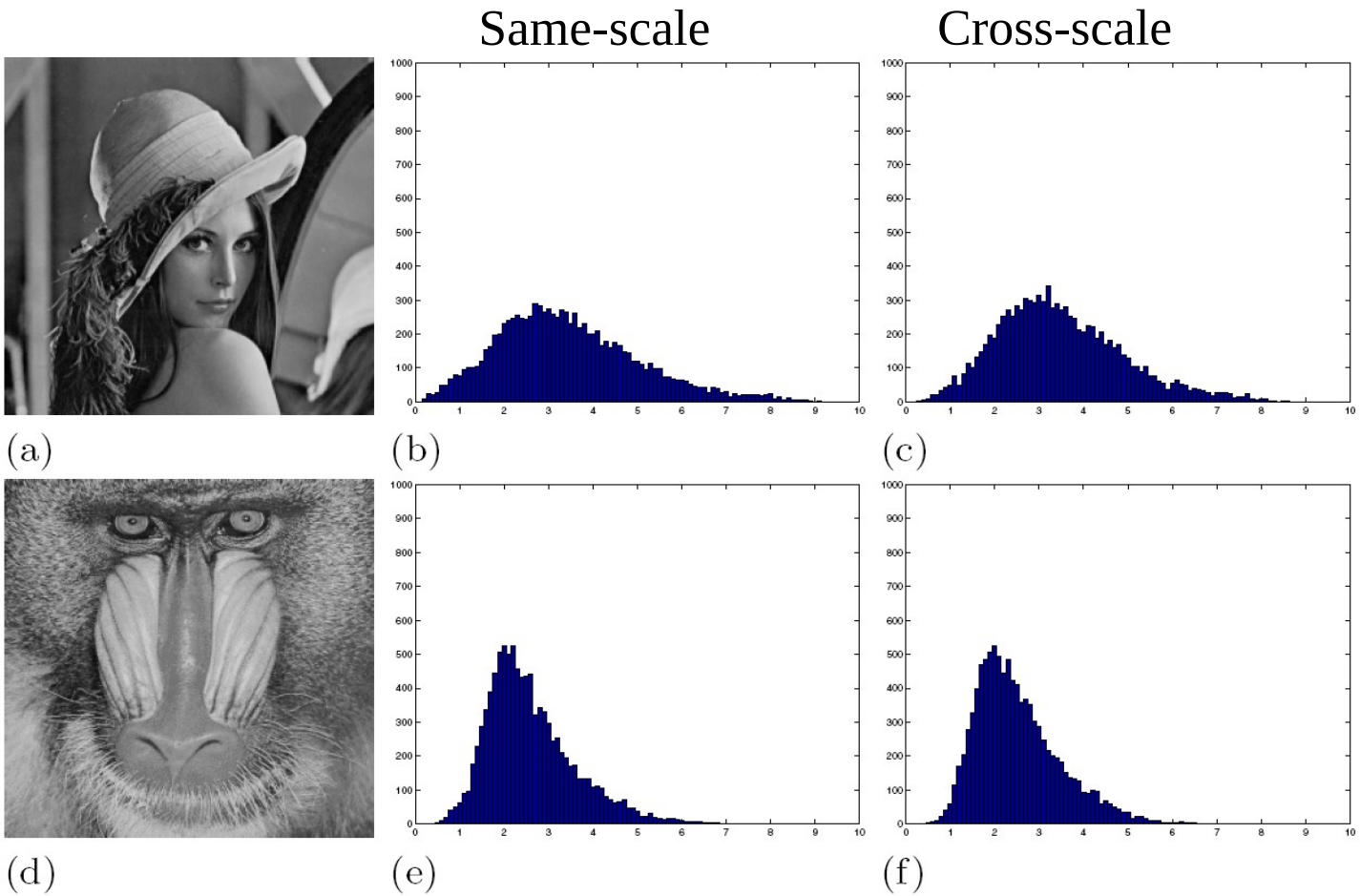
Solving the inverse problem of image zooming using “self-examples”,

in which we made use of cross-scale self-similarity.

Distribution of Neighborhood Distances at Various Scales

256x256 images

Patch-size 13x13



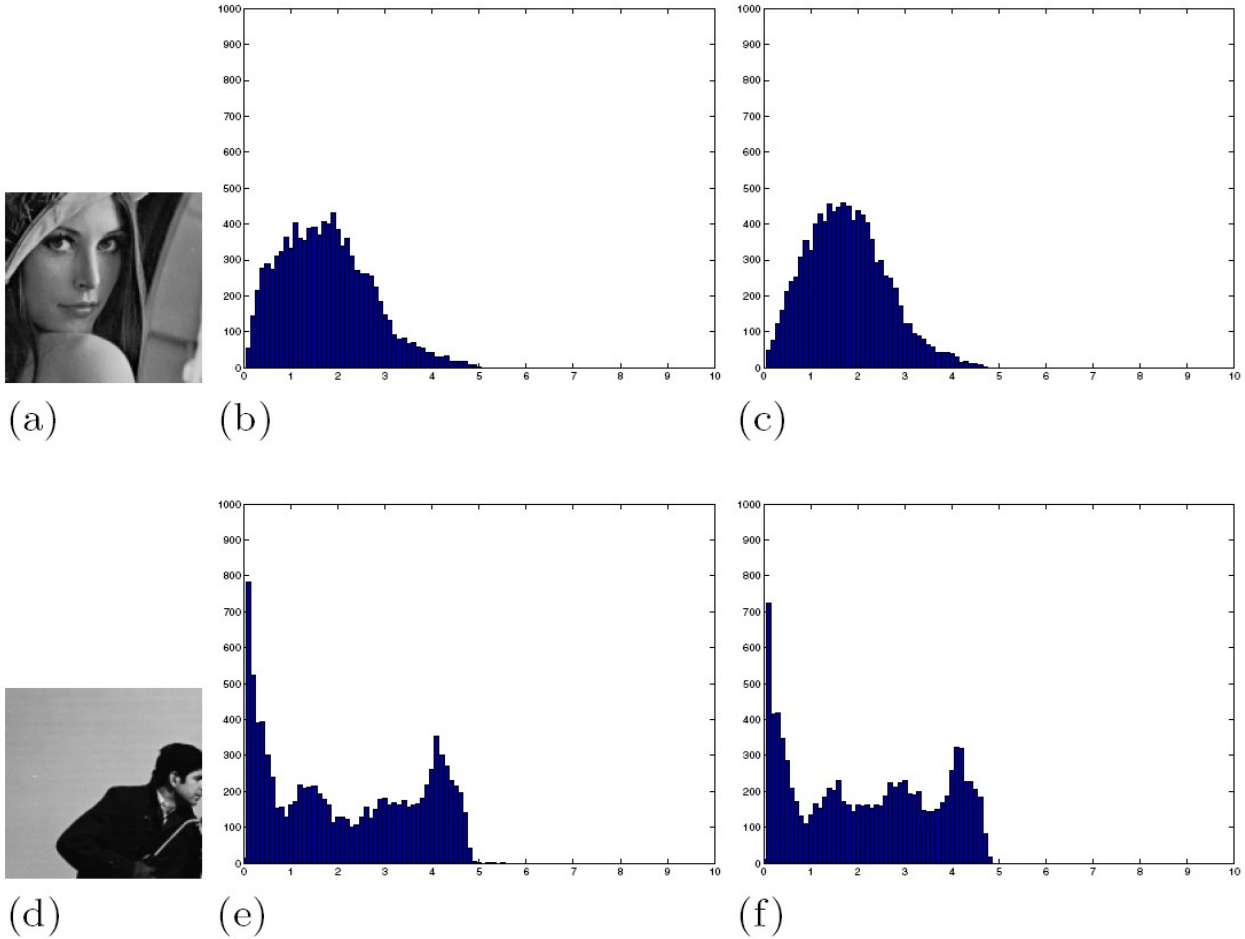
Distribution of Neighborhood Distances at Various Scales

Same-scale

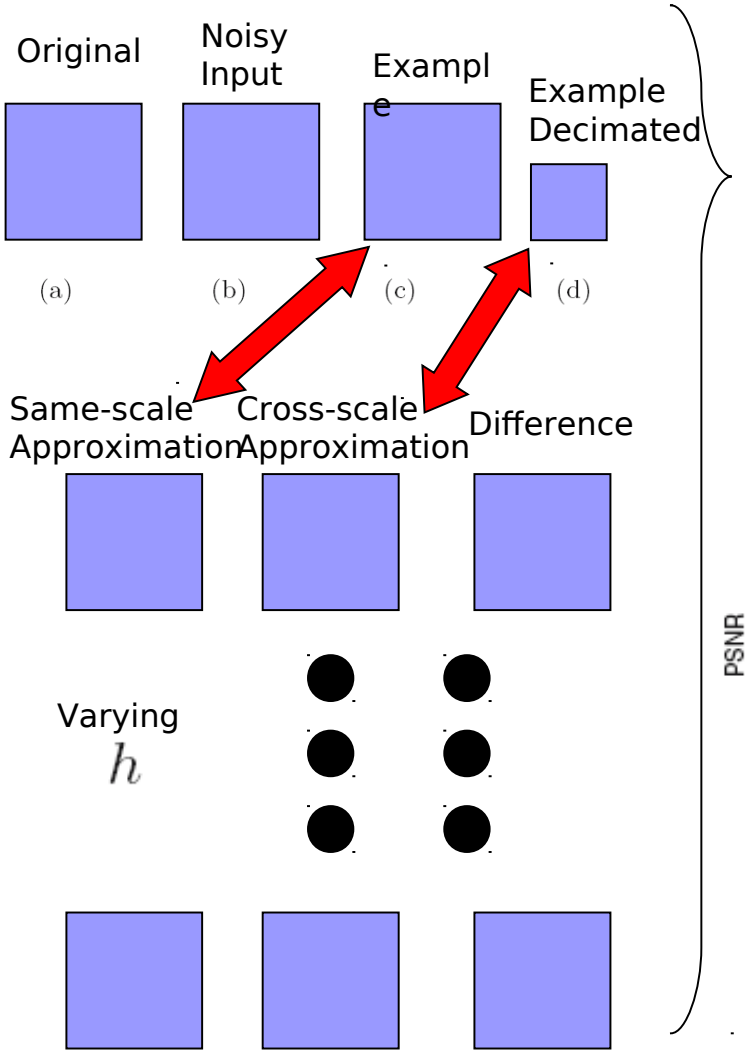
Cross-scale

128x128 images

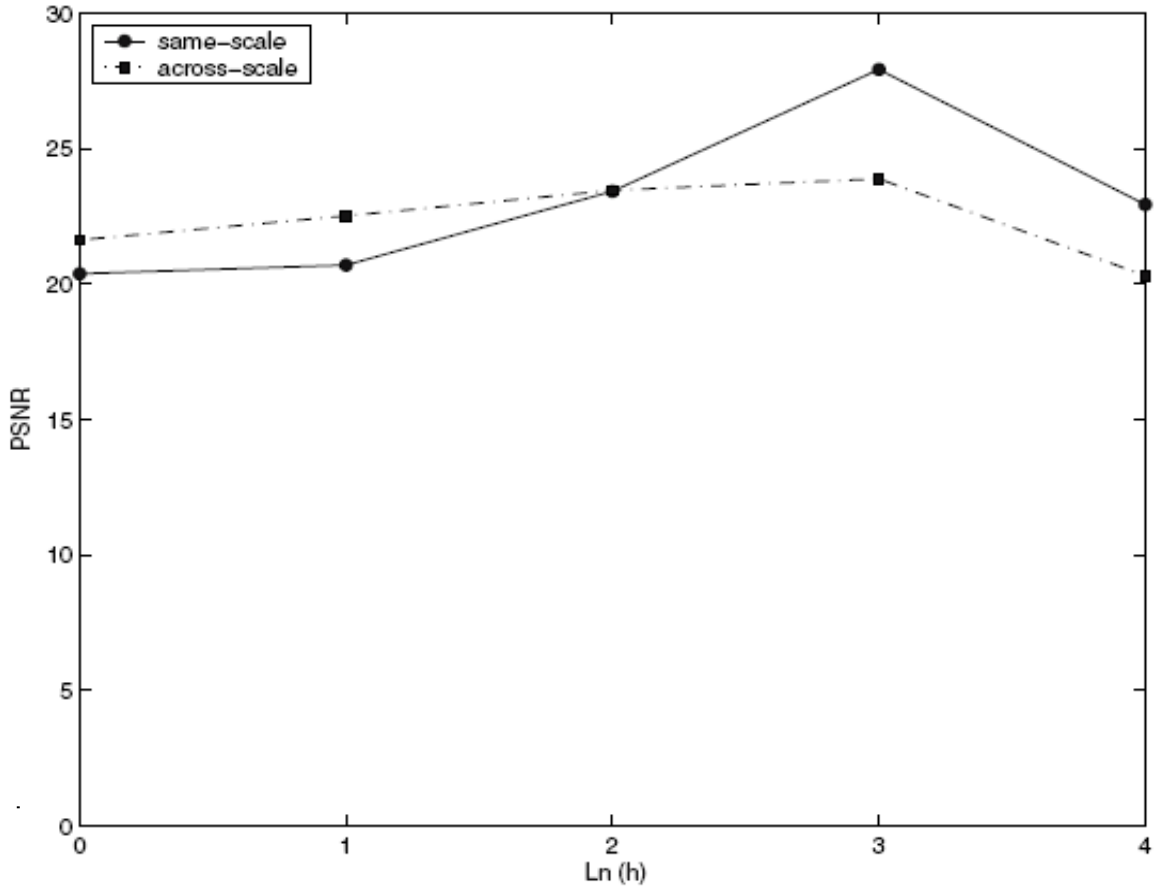
Patch-size 7x7



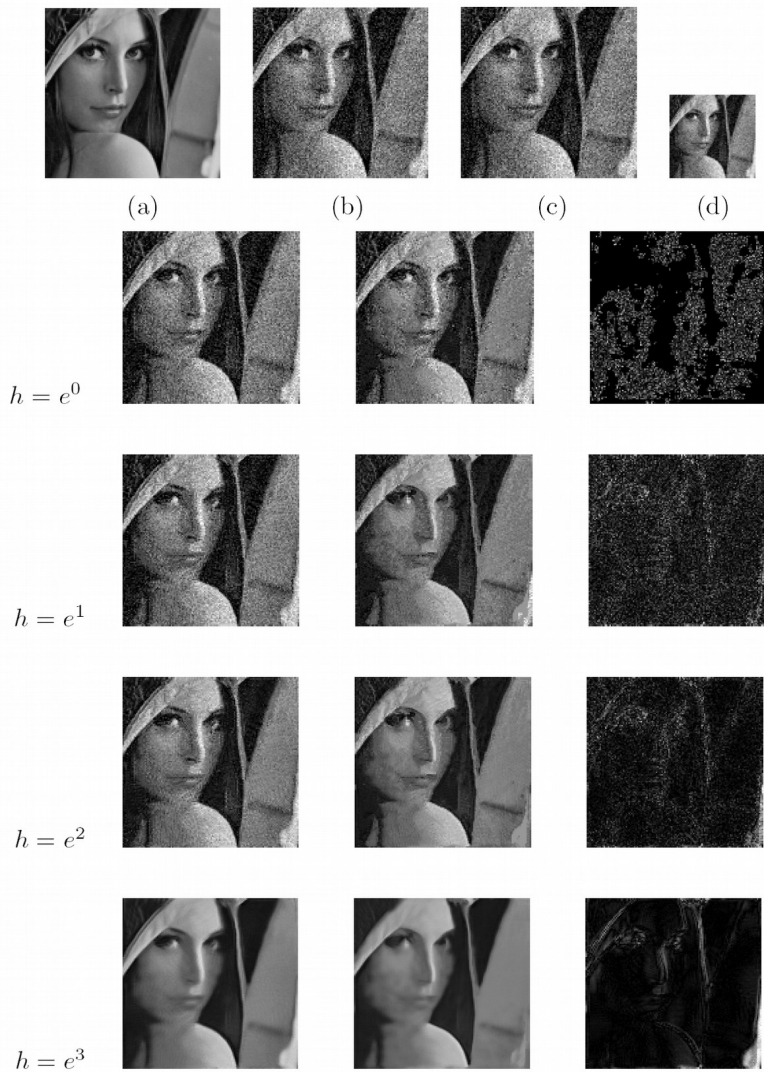
Same-scale vs. Cross-scale Approximations

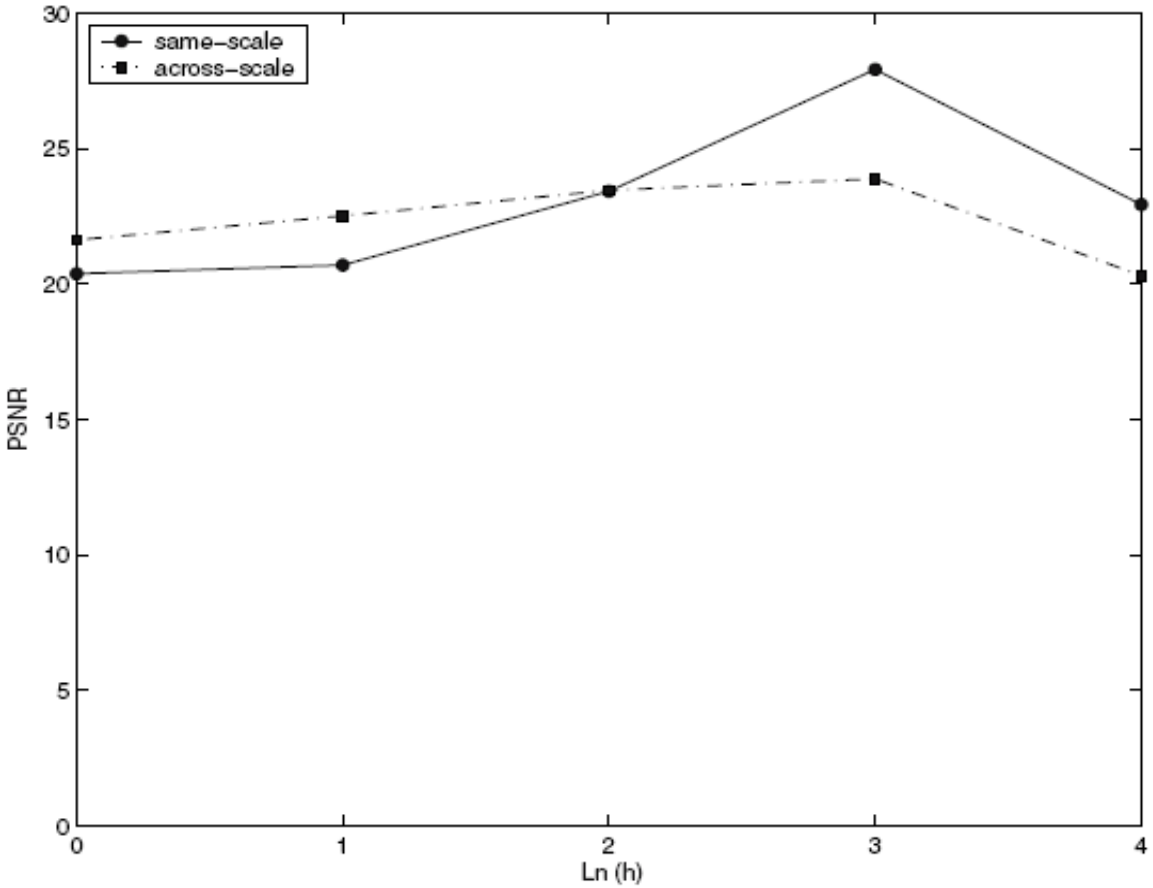


Template used for the following experiments

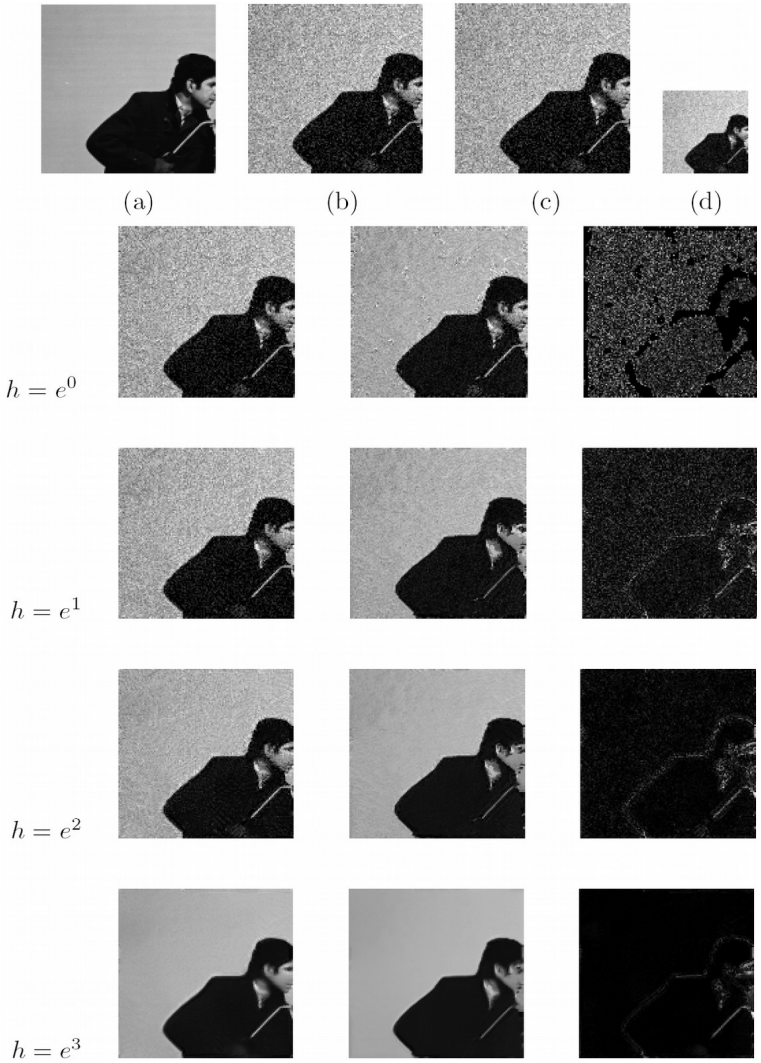


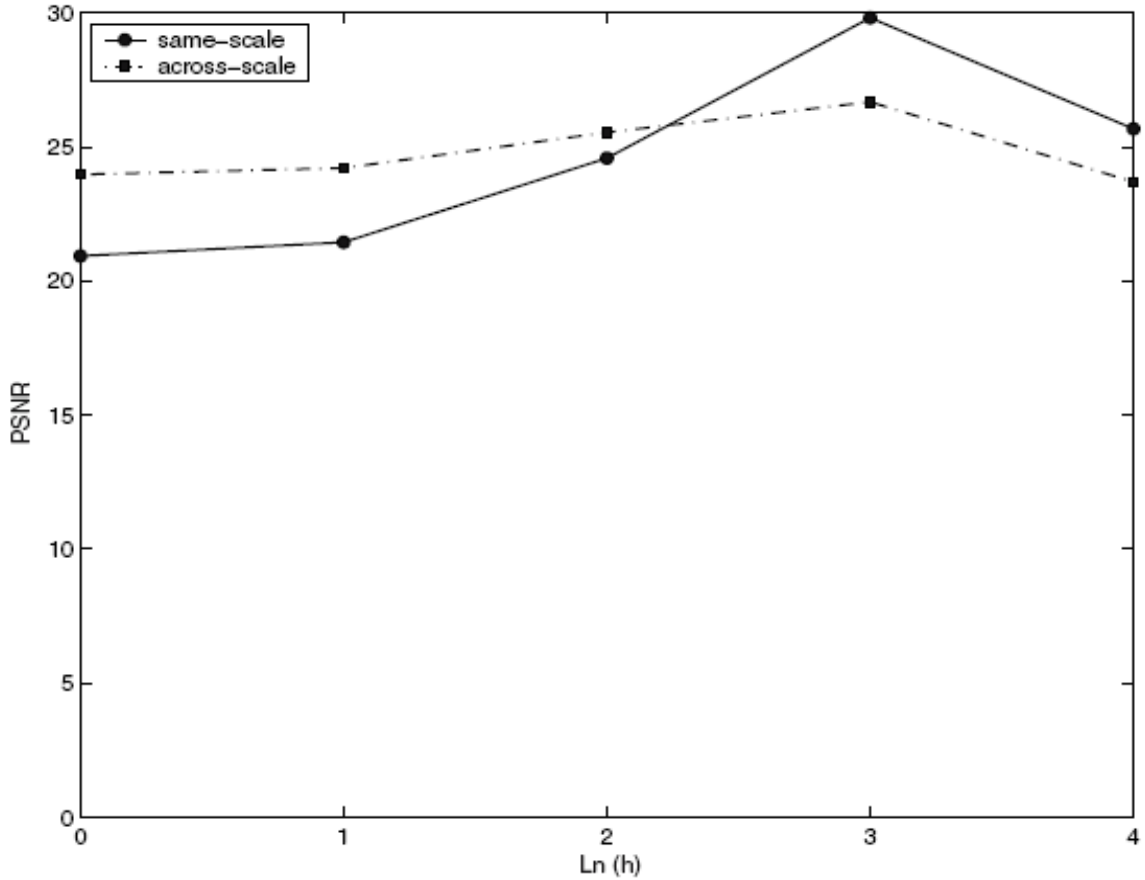
Same-scale vs. Cross-scale Approximations





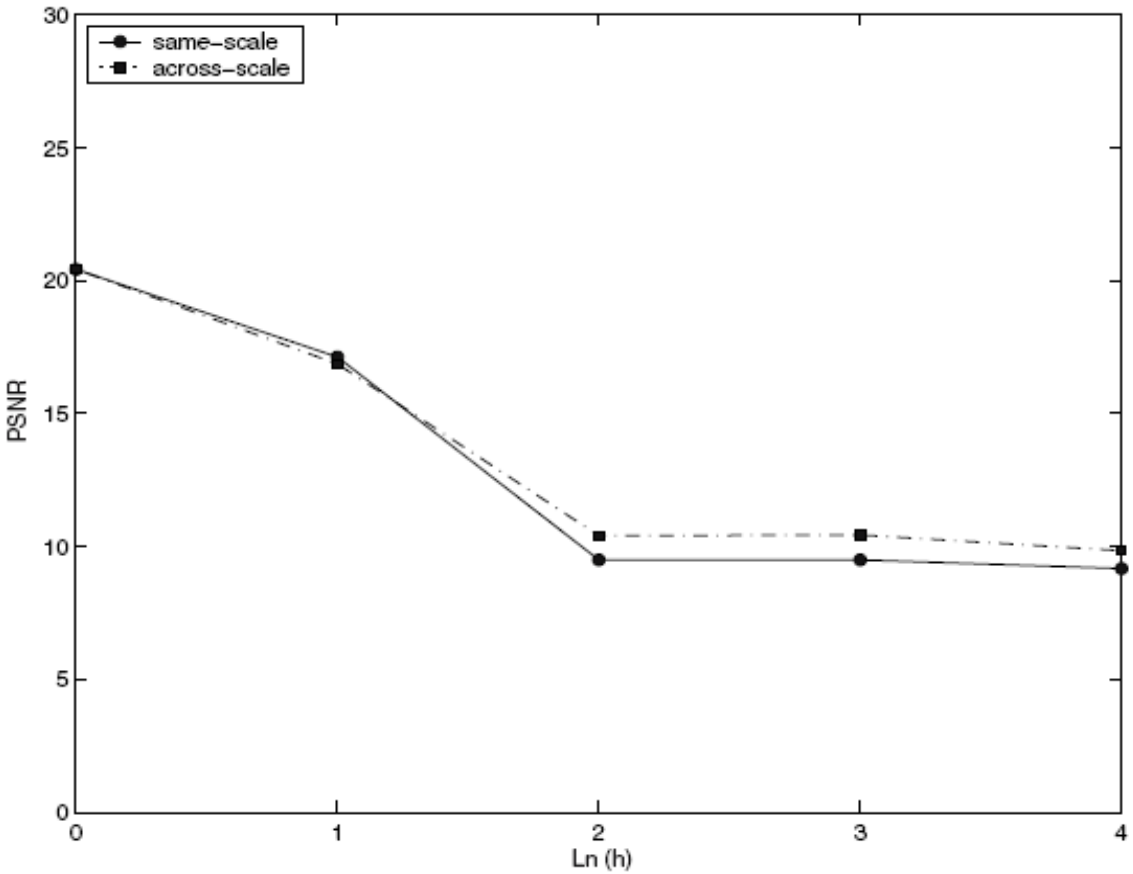
Same-scale vs. Cross-scale Approximations



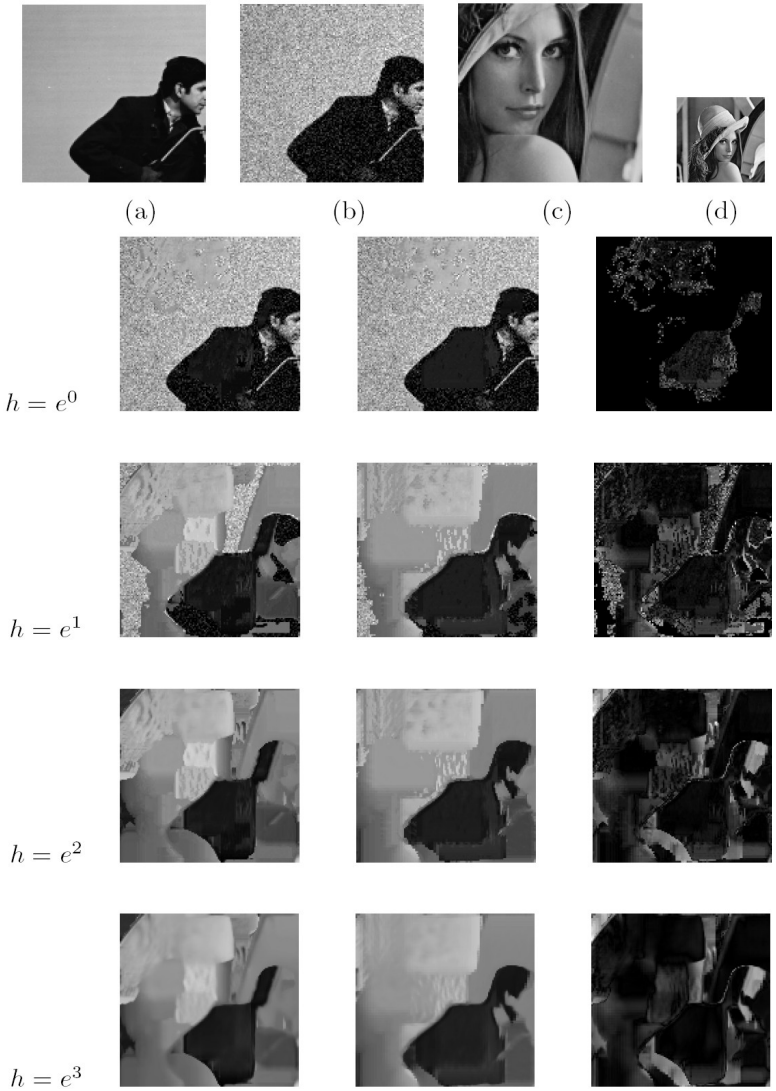


Same-scale vs. Cross-scale Approximations

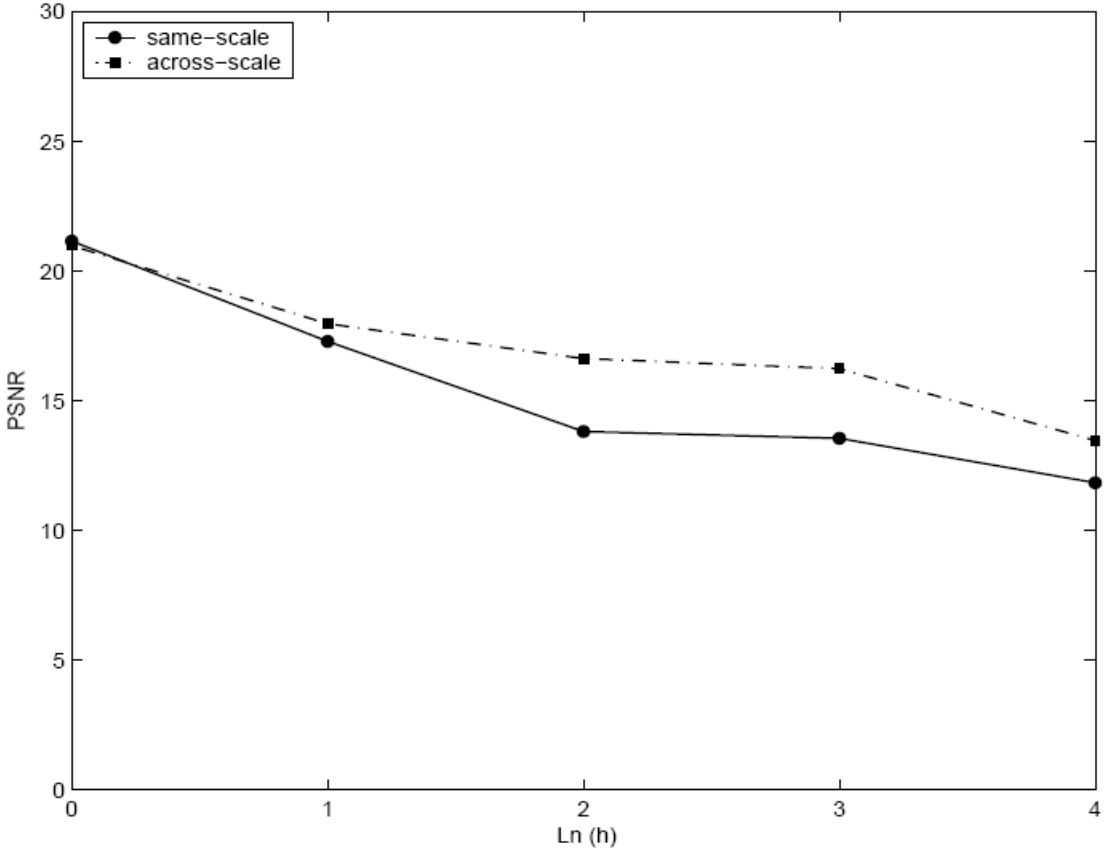
Irrelevant Example being used



Same-scale vs. Cross-scale Approximations



Irrelevant Example being used



Conclusions

- We made use of cross-scale similarity consistent with fractal-based methods, yet gave a generalization of the NL-means denoising algorithm in a natural fashion.
- Cross-scale patches are also “relevant” in approximating the image, as opposed to patches taken from some “irrelevant” image.
- This gives more insight that why examples from across-scales in an image can be used to solve the problem of single-frame image zooming using self-examples. (ICIAR `07)
- The question of “What example image to choose for a given observation in general?” remains open.