Structural similarity-based approximation of signals and images using orthogonal bases

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Introduction

Our investigation is centered around the so-called **structural similarity** (SSIM) index, $S(\mathbf{x}, \mathbf{y})$, a measure of the **visual distortion** between two images, $\mathbf{x}$ and $\mathbf{y}$.

1. It is well known that the well-known L^2 distance between $\mathbf{x}$ and $\mathbf{y}$, i.e.,

$$d_2(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2 \qquad \text{(MSE or RMSE)},$$

   is not necessarily a good measure of their **visual closeness/distance**.

2. For this reason, there has been much interest in developing **image quality metrics** that provide a better measure of visual closeness/distance.

3. SSIM is one such example:

   Z. Wang, A.C. Bovik, H.R. Sheikh and E.P. Simoncelli, "Image quality assessment: From error visibility to structural similarity," IEEE Trans. Image Processing, **13**(4), 600-612 (2004).

4. SSIM has been used in a wide range of applications: image compression, restoration, fusion, watermarking, video streaming, etc..

5. But much less has been done on using SSIM as an optimization criterion in the design and tuning of image processing algorithms and systems.

6. In this paper, we "step back" in an effort to understand SSIM-based approximation mathematically.

7. Specifically, we compare **optimal SSIM-based approximation** using orthogonal functions to the well-known result of **optimal L^2-based approximation**.

The inability of L^2 (and ability of SSIM) to measure visual closeness/distance

(a) *Einstein* test image $\mathbf{x}$ (upper left). All other images $\mathbf{y}_i$ are distorted versions. L^2 distance (MSE) between images (b)-(g) and $\mathbf{x}$ are roughly the same, i.e., $\|\mathbf{x} - \mathbf{y}_i\| \approx r$, constant. Visually, which of (b)-(g) is (i) best, (ii) worst? Images (g)-(l) illustrate effectiveness of complex-wavelet form of SSIM to detect translational distortion. From Wang *et al.* (2004).

Original formulation of SSIM index

Given two N-dimensional signals/images (or corresponding patches),

$$\mathbf{x} = (x_1, \cdots, x_N) \quad \text{and} \quad \mathbf{y} = (y_1, \cdots, y_N),$$

SSIM examines similarities between their **luminance**, **contrast** and **structure**.

1. For **luminance**, $l(\mathbf{x}, \mathbf{y})$, use mean values, e.g.,

$$\bar{\mathbf{x}} = \frac{1}{N} \sum_{i=1}^{N} x_i.$$

2. For **contrast**, $c(\mathbf{x}, \mathbf{y})$, use variances, e.g.,

$$s_{\mathbf{x}}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2.$$

3. For **structure**, $s(\mathbf{x}, \mathbf{y})$, use normalized signals (unit standard deviation), e.g.,

$$\mathbf{x}' = \frac{\mathbf{x} - \bar{\mathbf{x}}}{s_{\mathbf{x}}}$$

Then combine these components (in some way!) to yield an overall similarity measure, i.e.,

$$S(\mathbf{x}, \mathbf{y}) = F(l(\mathbf{x}, \mathbf{y}), c(\mathbf{x}, \mathbf{y}), s(\mathbf{x}, \mathbf{y})).$$

Original formulation of SSIM index (cont'd)

One more goal:

Incorporate, in some way, **Weber's model of perception**:

Visual sensitivity Σ to a change ΔI in greyscale intensity value I is well approximated by

$$\Sigma \approx \frac{\Delta I}{I}, \qquad \text{for} \;\; 0 < I_0 < I < I_1 < \infty.$$

This suggests that we should work with **ratios** of intensity values, means, etc..

Original formulation of SSIM index (cont'd)

Given two N-dimensional signals/images (or corresponding patches),

$$\mathbf{x} = (x_1, \cdots, x_N) \quad \text{and} \quad \mathbf{y} = (y_1, \cdots, y_N),$$

SSIM examines similarities between their **luminance**, **contrast** and **structure**.

1. For **luminance**, $l(\mathbf{x}, \mathbf{y})$, define
$$l(\mathbf{x}, \mathbf{y}) = \frac{2\bar{\mathbf{x}}\bar{\mathbf{y}} + \epsilon_1}{\bar{\mathbf{x}}^2 + \bar{\mathbf{y}}^2 + \epsilon_1}.$$
2. For **contrast**, $c(\mathbf{x}, \mathbf{y})$, define
$$c(\mathbf{x}, \mathbf{y}) = \frac{2s_{\mathbf{x}}s_{\mathbf{y}} + \epsilon_2}{s_{\mathbf{x}}^2 + s_{\mathbf{y}}^2 + \epsilon_2}.$$
3. For **structure**, $s(\mathbf{x}, \mathbf{y})$, compute correlation between normalized signals $\mathbf{x}'$ and $\mathbf{y}'$. But this is equal to correlation between $\mathbf{x}$ and $\mathbf{y}$, so define
$$s(\mathbf{x}, \mathbf{y}) = \frac{s_{\mathbf{xy}} + \epsilon_3}{s_{\mathbf{x}}s_{\mathbf{y}} + \epsilon_3},$$
where
$$s_{\mathbf{xy}} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{\mathbf{x}})(y_i - \bar{\mathbf{y}}).$$

The ϵ_i are **numerical stability coefficients** to prevent zero denominators. As well, they can model the deviation from Weber's model as intensities approach 0.

Original formulation of SSIM index (cont'd)

A closer look at the **luminance** function:

$$l(\mathbf{x}, \mathbf{y}) = \frac{2\bar{\mathbf{x}}\bar{\mathbf{y}} + \epsilon_1}{\bar{\mathbf{x}}^2 + \bar{\mathbf{y}}^2 + \epsilon_1}.$$

Note that

$$-1 \leq l(\mathbf{x}, \mathbf{y}) \leq 1.$$

If we assume that $\mathbf{x}$ is fixed and $\mathbf{y}$ is an approximation to $\mathbf{x}$, then rewrite $l(\mathbf{x}, \mathbf{y})$ as follows,

$$l(\mathbf{x}, \mathbf{y}) = \frac{2(\bar{\mathbf{y}}/\bar{\mathbf{x}}) + \epsilon_1'}{1 + (\bar{\mathbf{y}}/\bar{\mathbf{x}})^2 + \epsilon_1'},$$

showing dependence on ratio $\bar{\mathbf{y}}/\bar{\mathbf{x}}$, in accordance with Weber's model.

Connection between SSIM and L^2

First shown by Richter and Kim (2009)

Let $\mathbf{x}, \mathbf{y} \in \mathbf{R}^N$ and define

$$\mathbf{x}_0 = \mathbf{x} - \bar{\mathbf{x}}, \quad \mathbf{y}_0 = \mathbf{y} - \bar{\mathbf{y}} \quad \Rightarrow \quad \bar{\mathbf{x}}_0 = \bar{\mathbf{y}}_0 = 0.$$

Then

$$\begin{aligned} \|\mathbf{x}_0 - \mathbf{y}_0\|^2 &= \sum_{k=1}^{N} (x_{0,k} - y_{0,k})^2 \\ &= (N-1)[s^2_{\mathbf{x}_0} + s^2_{\mathbf{y}_0} - 2s_{\mathbf{x}_0\mathbf{y}_0}]. \end{aligned}$$

A little algebra ...

$$1 - S_0(\mathbf{x}_0, \mathbf{y}_0) = \frac{1}{N-1} \frac{\|\mathbf{x}_0 - \mathbf{y}_0\|^2}{s^2_{\mathbf{x}_0} + s^2_{\mathbf{y}_0}}.$$

The function,

$$0 \leq 1 - S_0(\mathbf{x}_0, \mathbf{y}_0) \leq 2,$$

is a kind of **variance-weighted** L^2 distance.

Original formulation of SSIM index (cont'd)

Given two N-dimensional signals/images (or corresponding patches),

$$\mathbf{x} = (x_1, \cdots, x_N) \quad \text{and} \quad \mathbf{y} = (y_1, \cdots, y_N),$$

general form of SSIM index (Wang et al. 2004):

$$S(\mathbf{x}, \mathbf{y}) = [l(\mathbf{x}, \mathbf{y})]^{\alpha} [c(\mathbf{x}, \mathbf{y})]^{\beta} [s(\mathbf{x}, \mathbf{y})]^{\gamma},$$

where $\alpha, \beta, \gamma > 0$.

Simplified version that is often used (also in this study):

Set $\alpha = \beta = \gamma = 1$ and $\epsilon_3 = \epsilon_2/2$ to give

$$S(\mathbf{x}, \mathbf{y}) = S_1(\mathbf{x}, \mathbf{y})\, S_2(\mathbf{x}, \mathbf{y}) = \left[\frac{2\bar{\mathbf{x}}\bar{\mathbf{y}} + \epsilon_1}{\bar{\mathbf{x}}^2 + \bar{\mathbf{y}}^2 + \epsilon_1}\right] \left[\frac{2s_{\mathbf{xy}} + \epsilon_2}{s_{\mathbf{x}}^2 + s_{\mathbf{y}}^2 + \epsilon_2}\right]$$

Note:

$$-1 \leq S(\mathbf{x}, \mathbf{y}) \leq 1 \quad \text{and} \quad S(\mathbf{x}, \mathbf{y}) = 1 \;\; \text{iff} \;\; \mathbf{x} = \mathbf{y}.$$

$$\text{If} \;\; \bar{\mathbf{x}} = \bar{\mathbf{y}} \;\; \text{then} \;\; S_1(\mathbf{x}, \mathbf{y}) = 1$$

Main idea - Best approximation to a signal $\mathbf{x} \in \mathbf{R}^N$

In what follows:

- $\mathbf{x} \in \mathbf{R}^N$ denotes a given signal,
- $A \subset \mathbf{R}^N$ an appropriate subset of $\mathbf{R}^N$,
- $\mathbf{x} \approx \mathbf{y} \in A$, approximation to $\mathbf{x}$ in A.

Best approximation in A to $\mathbf{x}$

- In L^2 sense: $\mathbf{y}_{A,L^2} = \arg\min_{y \in A} \|\mathbf{x} - \mathbf{y}\|_2$
- In SSIM sense: $\mathbf{y}_{A,SSIM} = \arg\max_{y \in A} S(\mathbf{x}, \mathbf{y}) = \arg\min_{y \in A} [1 - S(\mathbf{x}, \mathbf{y})]$.

Special case $A = \mathbf{R}^N$:

$$\mathbf{y}_{A,L^2} = \mathbf{y}_{A,SSIM} = \mathbf{x}$$

In this study, we consider the simplest situation: A is spanned by a subset of orthonormal basis functions of $\mathbf{R}^N$

Let $\{\psi_0, \psi_1, \cdots, \psi_N\}$ denote a complete set of orthonormal basis functions of $\mathbf{R}^N$ such that:

1. $\bar{\psi}_0 \neq 0, \quad \bar{\psi}_k = 0, \quad 1 \leq k \leq N,$

2. $\bar{\psi}_0 = N^{-1/2}(1, 1, \cdots, 1)$, i.e., ψ_0 is "flat". In this case, $\bar{\psi}_0 = N^{-1/2}$.

These conditions accomodate the DCT as well as Haar multiresolution system on $\mathbf{R}^N$.

L^2-based expansion of $\mathbf{x}$ in this basis is well-known:

$$\mathbf{x} = \sum_{k=0}^{N-1} a_k \psi_k, \quad a_k = \langle \mathbf{x}, \psi_k \rangle, \quad 0 \leq k \leq N-1 \quad (a_n \textit{ are Fourier coefficients} \text{ of } \mathbf{x}).$$

It follows that

$$\bar{\mathbf{x}} = \sum_{k=0}^{N-1} a_k \bar{\psi}_k = a_0 \bar{\psi}_0 = a_0 N^{-1/2}.$$

Expansions of the approxmations $\mathbf{y}$ will be denoted as follows,

$$\mathbf{y} = \mathbf{y}(\mathbf{c}) = \sum_{k=0}^{N-1} c_k \psi_k.$$

Therefore,

$$\bar{\mathbf{y}} = c_0 N^{-1/2}.$$

Note that $a_0 = c_0$ implies that $\bar{\mathbf{x}} = \bar{\mathbf{y}}$, which yields maximum value of $S_1(\mathbf{x}, \mathbf{y}) = 1$.

Here we consider subsets A that are spanned by subsets of the $\{\psi_k\}_{k=0}^{N-1}$ basis functions which include the element ψ_0, i.e.,

$$A = \text{span}\,\{\psi_0, \psi_{\gamma(1)}, \cdots, \psi_{\gamma(M-1)}\}, \qquad \gamma(k) \in \{1, 2, \cdots, N-1\}, \quad \gamma(k) \neq \gamma(l) \text{ if } k \neq l.$$

For a given $1 < M < N$, we consider all possible subsets. The goal is to find the *optimal* M-dimensional subsets in (i) the L^2 and (ii) the SSIM sense.

Note: The inclusion of ψ_0 in A is motivated from the previous result, i.e., that $a_0 = c_0$ implies that $\bar{\mathbf{x}} = \bar{\mathbf{y}}$.

At this point it is not clear if the L^2 and SSIM approximations are optimized with the same M-dimensional subset.

L^2-optimal M-dimensional subset A (Easy)

Recall that

$$A = \text{span}\,\{\psi_0, \psi_{\gamma(1)}, \cdots, \psi_{\gamma(M-1)}\}, \quad \gamma(k) \in \{1, 2, \cdots, N-1\}, \quad \gamma(k) \neq \gamma(l) \text{ if } k \neq l.$$

Best approximation of $\mathbf{x}$ in subset A is

$$\mathbf{y}(\mathbf{c})_{A,L^2} = \text{Proj}_A\, \mathbf{x} = c_0\psi_0 + \sum_{k=1}^{M-1} c_{\gamma(k)}\psi_{\gamma(k)},$$

where

$$c_0 = a_0 = \langle \mathbf{x}, \psi_0 \rangle$$

and

$$c_{\gamma(k)} = a_{\gamma(k)} = \langle \mathbf{x}, \psi_{\gamma(k)} \rangle, \quad 1 \leq k \leq M-1.$$

Squared L^2 error of this approximation is

$$\|\mathbf{x} - \mathbf{y}(\mathbf{c})_{A,L^2}\|_2^2 = \|\mathbf{x}\|_2^2 - a_0^2 - \sum_{k=1}^{M-1} a_{\gamma(k)}^2.$$

Optimal M-dimensional subset A is the one for which the $M-1$ Fourier coefficients $a_{\gamma(k)}$ have the largest magnitudes.

SSIM-optimal M-dimensional subset A

Recall that

$$A = \text{span}\,\{\psi_0, \psi_{\gamma(1)}, \cdots, \psi_{\gamma(M-1)}\}, \qquad \gamma(k) \in \{1, 2, \cdots, N-1\}, \quad \gamma(k) \neq \gamma(l) \text{ if } k \neq l.$$

Proposition 2 of paper: The subset A is the same subset as for the L^2 case, i.e., the one for which the $M-1$ Fourier coefficients have the largest magnitudes. However:

The coefficients c_k of the optimal SSIM-based approximation in A, i.e.,

$$\mathbf{y}(\mathbf{c})_{A,SSIM} = \sum_{k=0}^{M-1} c_k \psi_{\gamma(k)},$$

are given by

$$c_0 = a_0, \qquad c_{\gamma(k)} = \alpha a_{\gamma(k)}, \quad 1 \leq k \leq M-1,$$

where the scaling coefficient α is given by

$$\alpha = \frac{-\epsilon_2 + \sqrt{\epsilon_2^2 + (\frac{4}{N-1}\sum_{k=1}^{M-1} a_{\gamma(k)}^2)(s_{\mathbf{x}}^2 + \epsilon_2)}}{\frac{2}{N-1}\sum_{k=1}^{M-1} a_{\gamma(k)}^2}.$$

Corollary: To obtain best SSIM approximation, first find the best M-dimensional L^2 approximation which includes the coefficient a_0, then construct the coefficients $c_{\gamma(k)}$ according to above prescription.

Note: When $M = N$, we have $\alpha = 1$ and $\mathbf{x} = \mathbf{y}(\mathbf{c})$, as expected.

Sketch of derivation

We express the structural similarity $S(\mathbf{x}, \mathbf{y}(\mathbf{c})$ in terms of the coefficients c_k. First, for notational convenience, and without loss of generality, assume that the last $N - M$ coefficients c_k are zeros, and drop the $\gamma(k)$ notation. After a little algebra,

$$s_{\mathbf{xy}} = \frac{1}{N-1} \sum_{k=1}^{M-1} a_k c_k, \qquad s_{\mathbf{y}}^2 = \frac{1}{N-1} \sum_{k=1}^{M-1} c_k^2.$$

First term in SSIM depends only on c_0:

$$S_1(\mathbf{x}, \mathbf{y}(\mathbf{c})) = \frac{2\bar{\mathbf{x}}\bar{\mathbf{y}} + \epsilon_1}{\bar{\mathbf{x}}^2 + \bar{\mathbf{y}}^2 + \epsilon_1} = \frac{2a_0c_0 + \epsilon_1}{a_0^2 + c_0^2 + \epsilon_1}.$$

Second term in SSIM $S_2(\mathbf{x}, \mathbf{y}(\mathbf{c}))$ is independent of c_0. Therefore we set $c_0 = a_0$ to maximize S_1, giving it the value of 1.

The SSIM function then becomes a function of remaining coefficients $c_1, \cdots, c_{M-1}$:

$$S(\mathbf{x}, \mathbf{y}(\mathbf{c})) = \frac{\frac{2}{N-1}\sum_{k=1}^{M-1} a_k c_k + \epsilon_2}{s_{\mathbf{x}}^2 + \frac{1}{N-1}\sum_{k=1}^{M-1} c_k^2 + \epsilon_2}.$$

Now look for stationary points: Logarithmic differentiation yields the following partial derivatives:

$$\begin{aligned} \frac{\partial S}{\partial c_k} &= S\left[\frac{2\bar{\mathbf{x}}}{2\bar{\mathbf{x}}\bar{\mathbf{y}} + \epsilon_1}\frac{\partial \bar{\mathbf{y}}}{\partial c_k} + \frac{2}{2s_{\mathbf{xy}} + \epsilon_2}\frac{\partial s_{\mathbf{xy}}}{\partial c_k}\right. \\ &\qquad \left. - \frac{2\bar{\mathbf{y}}}{\bar{\mathbf{x}}^2 + \bar{\mathbf{y}}^2 + \epsilon_1}\frac{\partial \bar{\mathbf{y}}}{\partial c_k} - \frac{1}{s_{\mathbf{x}}^2 + s_{\mathbf{y}}^2 + \epsilon_2}\frac{\partial s_{\mathbf{y}}^2}{\partial c_k}\right]. \end{aligned}$$

After some additional (yet simple) algebra, we obtain the following conditions for a stationary point,

$$\frac{\partial S}{\partial c_k} = \frac{S}{N-1}\left[\frac{2a_k}{2s_{\mathbf{xy}}+\epsilon_2} - \frac{2c_k}{s_{\mathbf{x}}^2+s_{\mathbf{y}}^2+\epsilon_2}\right] = 0, \quad 1 \le k \le M-1.$$

If $a_p = 0$ for any $1 \le p \le M-1$, then $c_p = 0$. Otherwise, we have that

$$\frac{a_k}{c_k} = \frac{2s_{\mathbf{xy}}+\epsilon_2}{s_{\mathbf{x}}^2+s_{\mathbf{y}}^2+\epsilon_2}, \qquad 1 \le k \le M-1. \quad (*)$$

Note that the RHS of each equation is independent of k, implying that

$$\frac{a_1}{c_1} = \frac{a_2}{c_2} = \cdots = \frac{a_{M-1}}{c_{M-1}} = C \ \ \text{(constant)}.$$

Hence

$$c_k = \alpha a_k \quad \text{for } 1 \le k \le M-1, \quad (**)$$

where $\alpha = 1/C$. Of course, we have to solve for C.

We now rewrite Eq. (*) as follows,

$$(2s_{\mathbf{xy}}(\mathbf{c})+\epsilon_2)c_k = (s_{\mathbf{x}}^2 + s_{\mathbf{y}}^2(\mathbf{c})+\epsilon_2)a_k,$$

and employ (**) and the expansions in c_k to arrive at the following quadratic equation in α:

$$\frac{2\alpha^2}{N-1}\sum_{k=1}^{M-1} a_k^2 + \alpha\epsilon_2 = s_{\mathbf{x}}^2 + \frac{\alpha^2}{N-1}\sum_{k=1}^{M-1} a_k^2 + \epsilon_2.$$

Quadratic equation in scaling coefficient α:

$$\frac{2\alpha^2}{N-1}\sum_{k=1}^{M-1} a_k^2 + \alpha\epsilon_2 = s_{\mathbf{x}}^2 + \frac{\alpha^2}{N-1}\sum_{k=1}^{M-1} a_k^2 + \epsilon_2.$$

Two roots (see paper for expression):

1. $\alpha_1 \geq 1$: local maximum - desired root: call it α.

2. $\alpha_2 \leq -1$: local minimum

Special case: $\epsilon_1 = \epsilon_2 = 0$

$$\alpha = \left[\sum_{k=1}^{N-1} a_k^2\right]^{1/2} \left[\sum_{k=1}^{M-1} a_k^2\right]^{-1/2} \geq 1.$$

Substitute

$$c_0 = a_0, \quad c_k = \alpha a_k$$

into expression for SSIM:

$$S_{\max}(\mathbf{x}, \mathbf{y}(\mathbf{c})) = \frac{\frac{2\alpha}{N-1}\sum_{k=1}^{M-1} a_k^2 + \epsilon_2}{s_{\mathbf{x}}^2 + \frac{\alpha^2}{N-1}\sum_{k=1}^{M-1} a_k^2 + \epsilon_2}.$$

From earlier quadratic equation, we obtain the most interesting result,

$$S_{\max}(\mathbf{x}, \mathbf{y}(\mathbf{c})) = \frac{1}{\alpha}.$$

Eventually we find that

$$S_{\max}(\mathbf{x}, \mathbf{y}(\mathbf{c})) = \frac{\epsilon_2 + \sqrt{\epsilon_2^2 + (\frac{4}{N-1}\sum_{k=1}^{M-1} a_k^2)(s_{\mathbf{x}}^2 + \epsilon_2)}}{2(s_{\mathbf{x}}^2 + \epsilon_2)}. \tag{1}$$

From this expression, it is clear that the maximum possible value of $S_{\max}$ is achieved if the $M-1$ Fourier coefficients a_k with the largest magnitudes are employed in the summation. □

Summary

Recall that

$$A = \text{span}\,\{\psi_0, \psi_{\gamma(1)}, \cdots, \psi_{\gamma(M-1)}\}, \qquad \gamma(k) \in \{1, 2, \cdots, N-1\}, \quad \gamma(k) \neq \gamma(l) \text{ if } k \neq l.$$

Best L^2-based approximation to $\mathbf{x}$ in A is

$$\mathbf{y}_{A,L^2} = a_0\psi_0 + \sum_{k=1}^{N-1} a_{\gamma(k)}\psi_k, \quad a_\gamma(k) = \langle \mathbf{x}, \psi_{\gamma(k)} \rangle, \quad 1 \leq k \leq M-1.$$

Optimal subset A is the one for which the $M-1$ Fourier coefficients $a_{\gamma(k)}$ have the largest magnitudes.

Best SSIM-based approximation to $\mathbf{x}$ is obtained from same subset A,

$$\mathbf{y}_{A,SSIM} = a_0\psi_0 + \sum_{k=1}^{N-1} c_{\gamma(k)}\psi_k,$$

where

$$c_\gamma(k) = \alpha a_{\gamma(k)}, \quad 1 \leq k \leq M-1.$$

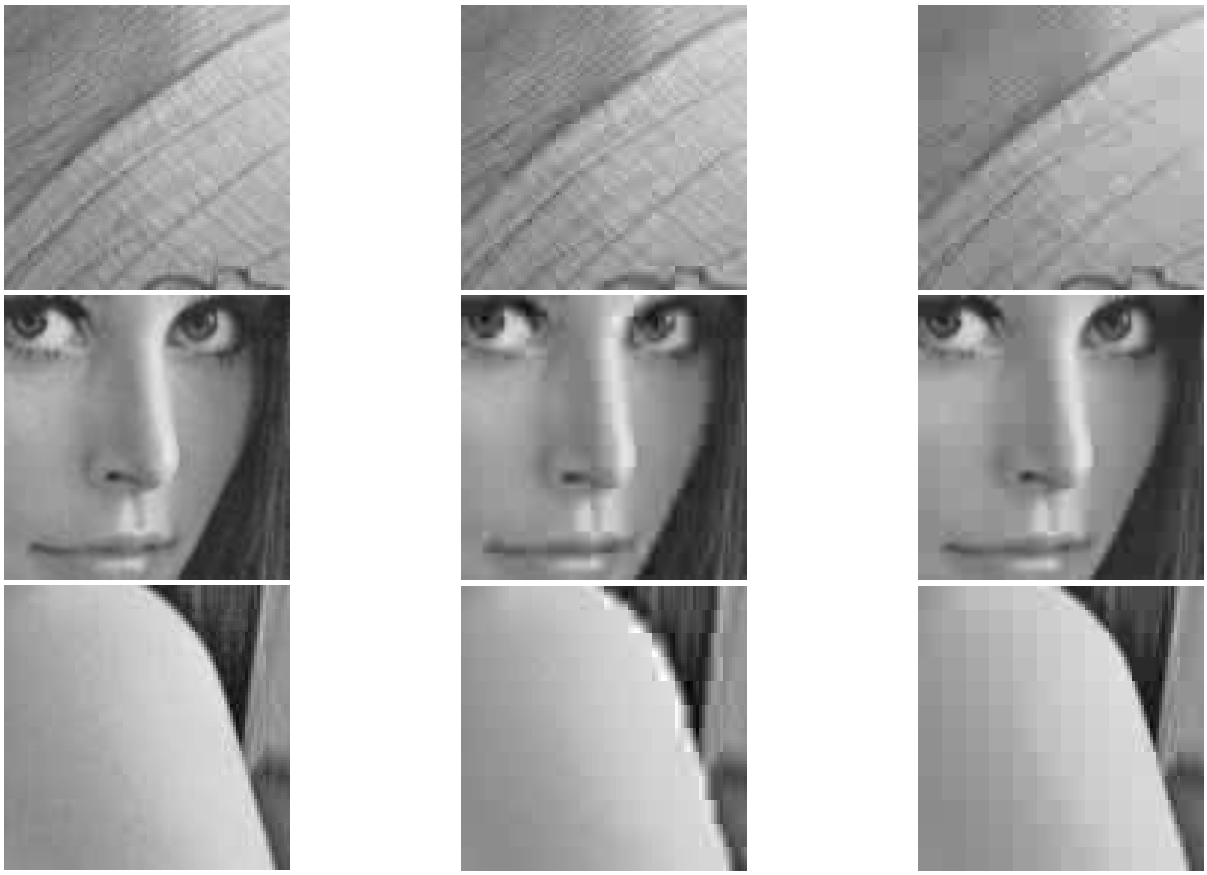
The scaling coefficient α is obtained from the Fourier coefficients a_k

1. Since $c_0 = a_0$ in both cases, it follows that

$$\bar{\mathbf{x}} = \bar{\mathbf{y}}_{A,L^2} = \bar{\mathbf{y}}_{A,SSIM}.$$

2. Since $\alpha > 1$ for $M < N$, $\bar{\mathbf{y}}_{A,SSIM}$ represents a **contrast-enhanced** version of $\mathbf{y}_{A,L^2}$.

Simple experiment: SSIM-based image reconstruction from a constrained number of DCT coefficients



Original (Lena) | SSIM-optimized | L^2-optimized

Figure 1: First column: Patches of the original *Lena* image. Second column: Patches of the BSSIM-optimized image. Third column: Patches of the L^2-optimized image.