Fulkerson 100 program with abstracts of invited talks

Wednesday, July 17

9-9:45am **Gérard Cornuéjols**

Title: Blocking theory and the replication conjecture

Abstract: I had the privilege of taking Ray Fulkerson's course on blocking and antiblocking theory when I was a student at Cornell University 50 years ago. The course covered balanced matrices and perfect graphs, and it had a profound influence on my career. Analogous to perfection in antiblocking theory is the notion of "the packing property" in blocking theory. A key insight on perfect graphs is the famous replication lemma proved by Laci Lovasz in 1972. In 1993, Michele Conforti and I proposed an analogous replication conjecture when the packing property holds. This conjecture is still open. This talk covers some recent developments related to the replication conjecture. These results are joint with Ahmad Abdi.

10:10-10:55am **Dan Král**

Title: Matroid depth and width parameters

Abstract: Depth and width parameters of graphs play a crucial role in algorithmic and structural graph theory, in particular, they are notions of fundamental importance in the theory of graph minors and in fixed parameter complexity. For example, the celebrated theorem of Courcelle asserts that every monadic second order property can be tested in polynomial time when inputs are restricted to classes of graphs of bounded tree-width. Another important graph parameter is tree-depth, which appears in many algorithmic and combinatorial contexts related to structural sparsity.
In this talk, we will survey structural and algorithmic results concerning matroid analogues of tree-depth and tree-width of graphs, and discuss their connections to discrete optimization. In particular, we will discuss structural properties of matroids, when viewed as combinatorial objects, related to matroid depth and width decompositions, and algorithmic consequences of such structural properties including matroid based algorithms that can uncover a hidden Dantzig-Wolfe-like structure of input instances of integer programs.

The most recent results presented in the talk are based on joint work with Marcin Briański, Jacob Cooper, Timothy F. N. Chan, Martin Koutecký, Ander Lamaison, Kristýna Pekárková and Felix Schröder.

11:05-11:50am **Cynthia Vinzant**

Title: Log-concavity in matroids and beyond

Abstract: Log-concavity is an important feature of many functions and discrete sequences appearing across mathematics, including combinatorics, algebraic geometry, convex analysis, and optimization. In this talk I describe how functional log-concavity of multivariate polynomials can be harnessed to approximately count and approximately sample from many interesting combinatorial distributions. Notable examples include determinantal point processes and the uniform distribution on the spanning trees of a graph.

2:30-3:15pm

Ola Svensson

Title: Advancements in online edge coloring algorithms

Abstract: In this presentation, we explore recent advancements in online edge coloring, highlighting the use of martingales to get close to optimal solutions. Our focus will be on two recent results:
Firstly, we address the longstanding conjecture by Bar-Noy, Motwani, and Naor, showing that a $(1+(1))\Delta$-edge-coloring can be achieved online for graphs with maximum degree $\Delta=\omega(\log n)$. This result holds even in the challenging setting of adversarial edge arrivals, and the techniques also allow us to extend known results in list edge coloring and local edge coloring.

Secondly, we consider the task of designing deterministic algorithms for online bipartite edge coloring. Contrary to the common belief that randomization is necessary to outperform the greedy algorithm, we give a deterministic approach in the vertex arrival model that achieves a competitive ratio of $e/(e-1)+o(1)$ for sufficiently large $\Delta=\omega(\log n)$.

Finally, we discuss several intriguing open questions related to these problems.

This is joint work with Joakim Blikstad, Radu Vintan, and David Wajc.

3:50-4:35pm

Jinyoung Park

Title: When sunflowers meet thresholds

Abstract: In this survey talk, we will discuss the connection between Alweiss-Lovett-Wu-Zhang's breakthrough on the Erdos-Rado Sunflower Conjecture and the recent developments around thresholds in probabilistic combinatorics.

4:45-5:30pm

Jim Geelen

Title: Average plane size

Abstract: In 1941, Melchior proved that, in a finite collection of points in the plane, not all on a single line, the average size of a spanned line is less than three. Here a spanned line is one that contains at least two distinct points from the collection, and the size of a line is the number of points that it contains from the collection. In joint work with Rutger Campbell and Matthew Kroeker, we have extended Melchior's result by considering the average plane-size of a finite collection of points in three-dimensional Euclidean space. We prove that the average is bounded above by an absolute constant unless the points all lie on a plane or they lie in two lines.

Thursday, July 18

9-9:45am **Maria Chudnovsky**

Title: Forbidding induced subgraphs: structure and algorithms

Abstract: Tree decompositions are a powerful tool in both structural graph theory and graph algorithms. Many hard problems become tractable if the input graph is known to have a tree decomposition of bounded "width". Exhibiting a particular kind of a tree decomposition is also a useful way to describe the structure of a graph.

Tree decompositions have traditionally been used in the context of forbidden graph minors; studying them in connection with graph containment relations of more local flavor (such as induced subgraph or induced minors) is a relatively new research direction. In this talk we will discuss recent progress in this area, touching on both the classical notion of bounded tree-width, and concepts of more structural flavor.

10:10-10:55am **Penny Haxell**

Title: Independent transversals, topology and resource allocation

Abstract: An independent transversal in a vertex-partitioned graph is an independent set that consists of exactly one vertex from each part of the partition. Certain discrete optimization problems involving fair allocation of resources have natural formulations in terms of the existence of an independent transversal in a suitably defined graph. We discuss both combinatorial and topological criteria for the existence of independent transversals in this setting, and how they lead to efficient approximation algorithms for resource allocation.

11:05-11:50am **Michel Goemans**

Title: From Ford-Fulkerson's network flows to information or linking flows

Abstract: Through their augmenting path algorithm, the max flow min cut theorem and their book, Ford and Fulkerson laid the foundations of classical network flows. As pointed out by Alan Hoffman, these can be viewed as a generalization of Menger's theorem for disjoint paths. Quite naturally, all these notions can be generalized to a matroid setting, and this has lead to Schrijver's linking systems and flows, which have found direct applications in information theory. This is essentially matroid intersection or partition/union reformulated, but is lesser known than the classical flow setting. In this talk, I will survey these notions. As an aside, I will also show how to prove matroid intersection through a simple charging argument, and without total unimodularity.

2-2:45pm **Éva Tardos**

Title: Stability and learning in strategic games

Abstract: Over the last two decades we have developed good understanding how to quantify the impact of strategic user behavior on outcomes in many games (including traffic routing and online auctions) and showed that the resulting bounds extend to repeated games assuming players use a form of learning (no-regret learning) to adapt to the environment. We will review how this area evolved since its early days, and also discuss some of the new frontiers, including when repeated interactions have carry-over effects between rounds: when outcomes in one round effect the game in the future, as is the case in many applications.

In this talk, we study this phenomenon in the context of a game modeling queuing systems: routers compete for servers, where packets that do not get served need to be resent, resulting in a system where the number of packets at each round depends on the success of the routers in the previous rounds. In joint work with Jason Gaitonde, we analyze the resulting highly dependent random processes, and show bounds on the resulting budgeted welfare for auctions and the excess server capacity needed to guarantee that all packets get served in the queuing system despite the selfish (myopic) behavior of the participants. We will briefly mention work with Giannis Fikioris in a different game, repeated auction with budgets, where the same issue arises also.

2:55-3:40pm **Satoru Iwata**

Title: Finding maximum edge-disjoint paths between multiple terminals

Abstract: Let $G = (V, E)$ be a multigraph with a set T of terminals. A path in G is called a T-path if its ends are distinct vertices in T and no internal vertices belong to T. In 1978, Mader showed a characterization of the maximum number of edge-disjoint T-paths.

In this talk, we provide a combinatorial, deterministic algorithm for finding the maximum number of edge-disjoint T-paths. The algorithm adopts an augmenting path approach. More specifically, we utilize a new concept of short augmenting walks in auxiliary labeled graphs to capture a possible augmentation of the number of edge-disjoint T-paths. To design a search procedure for a short augmenting walk, we introduce blossoms analogously to the matching algorithm of Edmonds (1965). When the search procedure terminates without finding a short augmenting walk, the algorithm provides a certificate for the optimality of the current edge-disjoint T-paths. From this certificate, one can obtain the Edmonds-Gallai type decomposition introduced by Sebő and Szegő (2004). The algorithm runs in $O(|E|^2)$ time, which is much faster than the best known deterministic algorithm based on a reduction to linear matroid parity.

We also present a strongly polynomial algorithm for the maximum integer free multiflow problem, which asks for a nonnegative integer combination of T-paths maximizing the sum of the coefficients subject to capacity constraints on the edges.

This is joint work with Yu Yokoi.

Friday, July 19

9-9:45am **Ravi Kannan**

Title: The random separating hyperplane theorem and learning latent polytopes

Abstract: A number of "hidden variable problems" from Machine Learning can be formulated as a geometric problem which we call the Latent k-Polytope (LkP) problem: Given highly perturbed points from a latent k-vertex polytope, learn its vertices. Clustering, Mixed Membership Community models, Topic Models can all be formulated as LkP. Under natural assumptions, we devise an algorithm to solve LkP. Our main tool is a geometric theorem we prove - the Random separating hyperplane (RSH) theorem. It states: if a point x is at distance δ (diameter of) K from a k-polytope K, the probability that a random hyperplane separates x from K is at least a certain function of δ, k. RSH generalizes the classical separating hyperplane theorem for polytopes and relates to the Johnson-Lindenstrauss Random Projection theorem and is likely of independent interest.

Joint work with Chiranjib Bhattacharyya and Amit Kumar.

10:10-10:55am **Thomas Rothvoss**

Title: The subspace flatness conjecture and faster integer programming

Abstract: In a seminal paper, Kannan and Lov\'asz (1988) considered a quantity $\mu_{KL}(\Lambda, K)$ which denotes the best volume-based lower bound on the *covering radius* $\mu(\Lambda, K)$ of a convex body K with respect to a lattice $\backslash Lambda$. Kannan and Lov\'asz proved that $\mu(\Lambda, K) \leq \ n \cdot \mu_{KL}(\Lambda, K)$ and the Subspace Flatness Conjecture by Dadush (2012) claims an $O(\log n)$ factor suffices, which would match the lower bound from the work of Kannan and Lov\'asz.

We settle this conjecture up to a constant in the exponent by proving that $\mu(\Lambda, K) \leq O(\log^3 n) \cdot \mu_{KL} \ (\Lambda, K)$. Our proof is based on the Reverse Minkowski Theorem due to Regev and Stephens-Davidowitz (2017).

Following the work of Dadush (2012, 2019), we obtain a $(\log n)^{O(n)}$-time randomized algorithm to solve integer programs in n variables. Another implication of our main result is a near-optimal *flatness constant* of $O(n \log^3 n)$.

This is joint work with Victor Reis.

11:05-11:50am **Sam Fiorini**

Title: Integer programs with nearly totally unimodular matrices: the cographic case

Abstract: It is a notorious open question whether integer programs (IPs) with an integer coefficient matrix M whose subdeterminants are all bounded by a constant Δ in absolute value, can be solved in polynomial time. We answer this question in the affirmative if we further require that, by removing a constant number of rows and columns from M, one obtains a submatrix A that is the transpose of a network matrix.

Our approach focusses on the case where A arises from M after removing k rows only, where k is a constant. We achieve our result in two main steps, the first related to the theory of IPs and the second related to graph minor theory.

First, we derive a strong proximity result for the case where A is a general totally unimodular matrix: Given an optimal solution of the linear programming relaxation, an optimal solution to the IP can be obtained by finding a constant number of augmentations by circuit vectors.

Second, for the case where A is transpose of a network matrix, we reformulate the problem as a maximum constrained potential problem on a graph G. We observe that if G is 2-connected, then it has no rooted $K_{2,t}$-minor for $t = \Omega(k\Delta)$. We leverage this to obtain a tree-decomposition of G into highly structured graphs for which we can solve the problem locally. This allows us to solve the global problem via dynamic programming.

This is joint work with Manuel Aprile (U Padova), Gwenaël Joret (ULB), Stefan Kober (ULB), Michał Seweryn (ULB), Stefan Weltge (TU Munich) and Yelena Yuditsky (ULB).

2-2:45pm **Ken-ichi Kawarabayashi**

Title: Three-edge-coloring projective planar cubic graphs: a generalization of the Four Color Theorem

Abstract: We prove that every cyclically 4-edge-connected cubic graph that can be embedded in the projective plane, with the single exception of the Petersen graph, is 3-edge-colorable. In other words, the only (non-trivial) snark that can be embedded in the projective plane is the Petersen graph.

This implies that a 2-connected cubic (multi)graph that can be embedded in the projective plane is not 3-edge-colorable if and only if it can be obtained from the Petersen graph by replacing each vertex by a 2-edge-connected planar cubic (multi)graph. Here, a replacement of a vertex v in a cubic graph G is the operation that takes a 2-connected planar (cubic) multigraph H containing some vertex u of degree 3, unifying $G - v$ and $H - u$, and connecting the vertices in $N_G[v]$ in $G - v$ with the three neighbors of u in $H - u$ with 3 edges. Any graph obtained in such a way is said to be *Petersen-like*.

This result is a nontrivial generalization of the Four Color Theorem, and its proof requires a combination of extensive computer verification and computer-free extension of existing proofs on colorability. We also discuss some further extensions, including cubic graphs embeddable on the torus, and subcubic planar graphs.

3:30-4:15pm

(*Special Tutte Colloquium*) **Paul Seymour**

Title: Nearly-linear stable sets

Abstract: The Gyárfás-Sumner conjecture says that for every forest H and complete graph K, there exists c such that every $\{H,K\}$-free graph (that is, containing neither of H, K as an induced subgraph) has chromatic number at most c. This is still open, but we have proved that every $\{H,K\}$-free graph G has chromatic number at most $|G|^{o(1)}$.

Second, a “multibroom” is a graph obtained from a tree of radius two by subdividing (arbitrarily) the edges incident with the root of the tree. It is not known that all multibrooms satisfy the Gyárfás-Sumner conjecture, but we have proved that they satisfy it with “chromatic number” replaced by “fractional chromatic number”.

Third, fix a graph H and a clique K, and suppose that G is a K-free graph that contains no subdivision of H as an induced subgraph. The chromatic number of G need not be bounded, but we have proved that it is at most $|G|^{o(1)}$ (and the same is true if we only exclude K and subdivisions of H in which each edge is subdivided at most $O(\log |G|)$ times).

These results are all proved by finding linear or nearly-linear stable sets of vertices. We will survey this material and sketch some of the proofs (assuming it doesn't all fall down – these results were only proved a few days ago).

Joint work with Tung Nguyen and Alex Scott.