Demystifying and Generalizing BinaryConnect

Yao-Liang Yu
(Joint work with Tim Dockhorn, Eyyub Sari, Mahdi Zolnouri, Vahid Partovi Nia)

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UNIVERSITY OF WATERLOO

The “Big” Cost

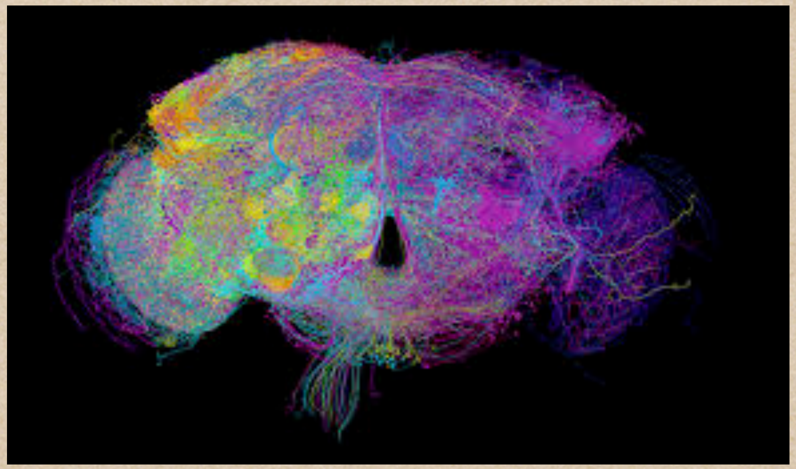
The "Big" Cost

big data

The "Big" Cost

big data

big model



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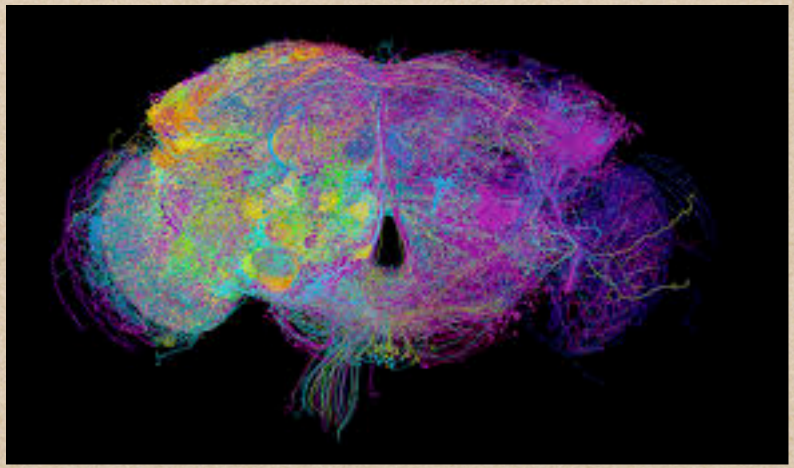

big computing

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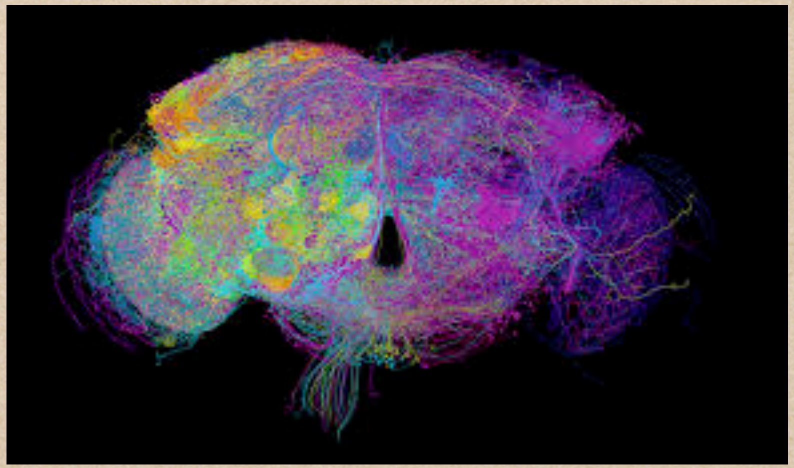

big computing

big consumption

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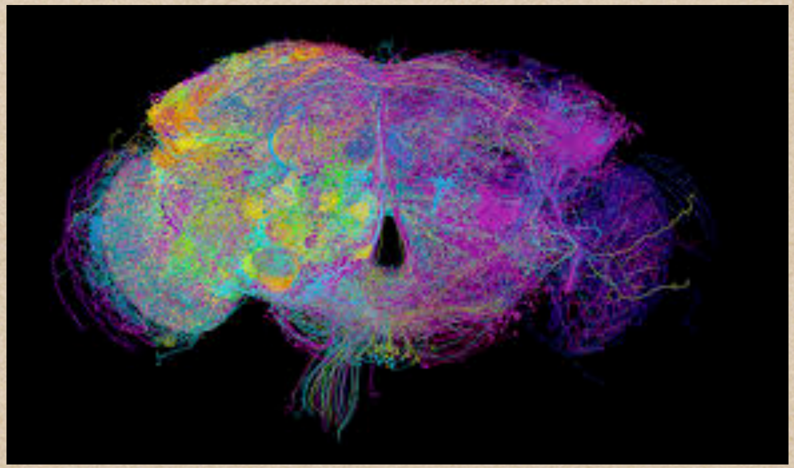


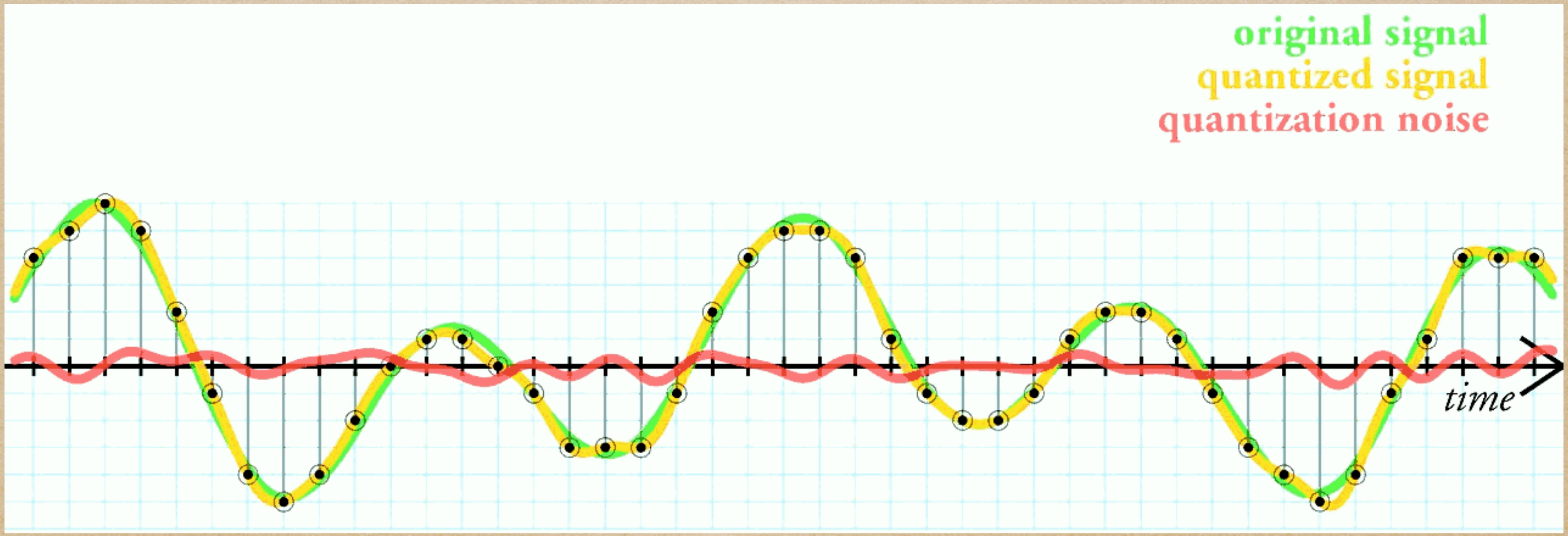
small devices

big computing

big consumption

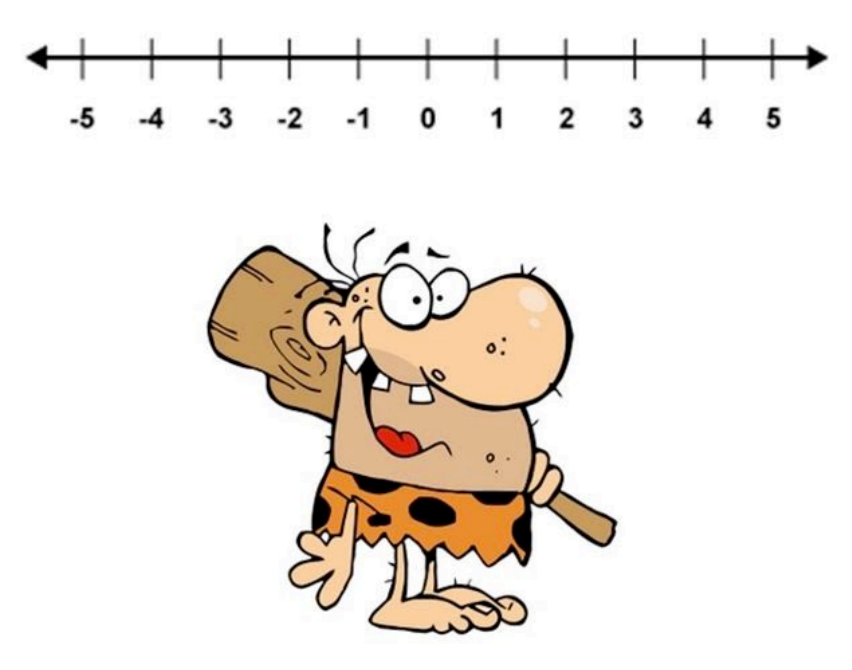
Quantization

- Full precision weights —> low precision
- Immediately reduced memory and energy
- Can also quantize activations/gradients/etc.



An Inherent Problem

- Real numbers do not really exist in physical world, or at least in current digital computers
- Create lots of headaches, a.k.a. numerical analysis
- We all deal with it, often by ignoring it and hoping things do not break…



Background

QNN Problem

$$\min_{\mathbf{w} \in Q} \ \ell(\mathbf{w})$$

- ℓ : loss we aim to minimize, e.g. cross-entropy as a surrogate for misclassification error
- Q: quantization set, usually consists of finitely many values, e.g. $Q = \{\pm 1\}^d$ for binary nets
- Solving QNN is very challenging, but we "just" aim to compete against the continuous network:

$$\min_{\mathbf{w}^* \in \mathbb{R}^d} \ \ell(\mathbf{w}^*)$$

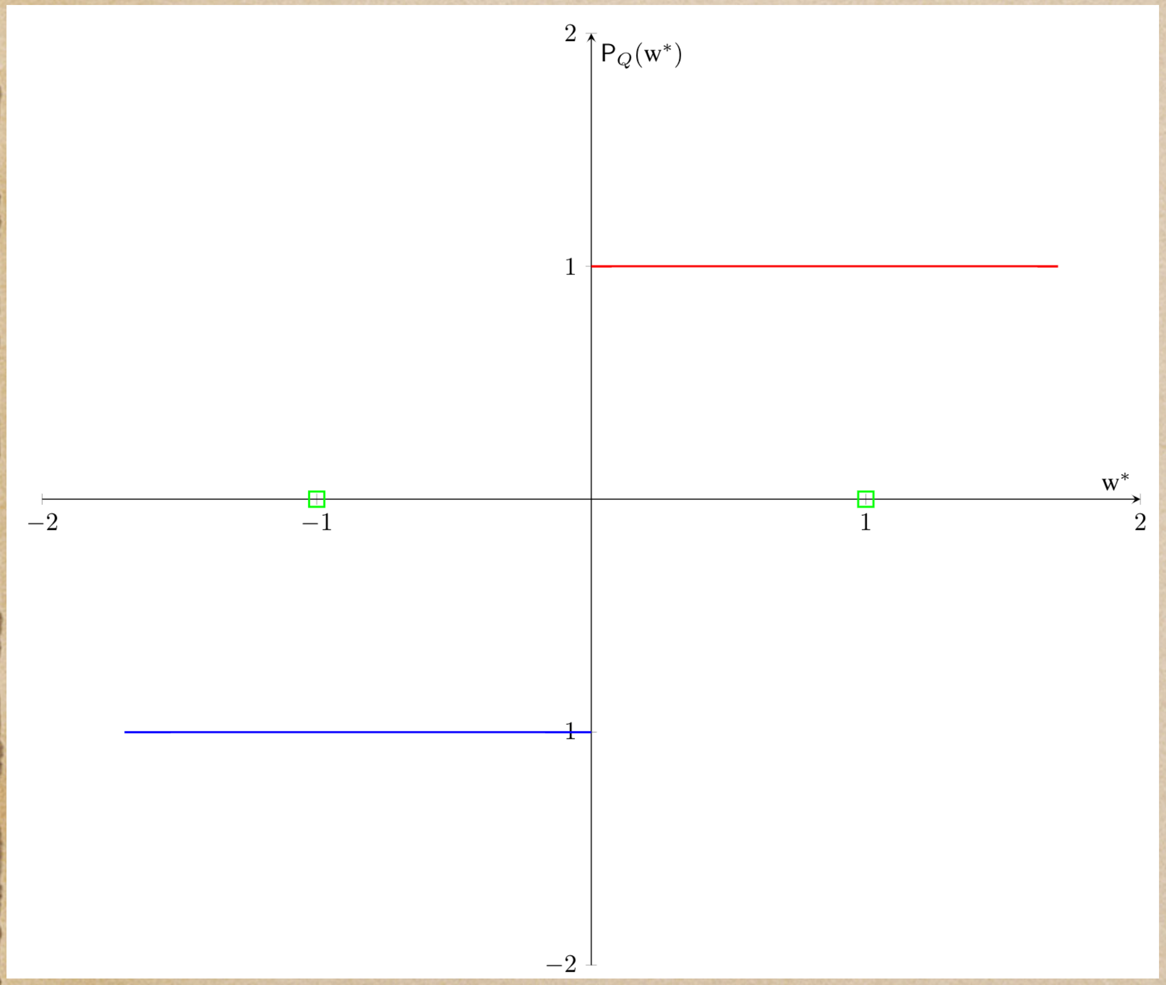
Binary Connect

$$\mathbf{w}^*_{t+1} = \mathbf{w}^*_t - \eta_t \widetilde{\nabla} \ell\left(\mathbf{P}(\mathbf{w}^*_t)\right)$$

- The quantizer P, e.g. the sign function, thresholds continuous weights $\mathbf{w}^*$ into discrete (binary) ones.
- Many other choices of P have been invented since.
- Setting P = id, we recover the usual SGD for training NNs.

- M. Courbariaux, Y. Bengio, and J.-P. David. *BinaryConnect: Training Deep Neural Networks with Binary Weights during Propagations*. NeurIPS (2015).
- I. Hubara, M. Courbariaux, D. Soudry, R. El-Yaniv, and Y. Bengio. *Quantized Neural Networks: Training Neural Networks with Low Precision Weights and Activations*. JMLR (2017).

Projector



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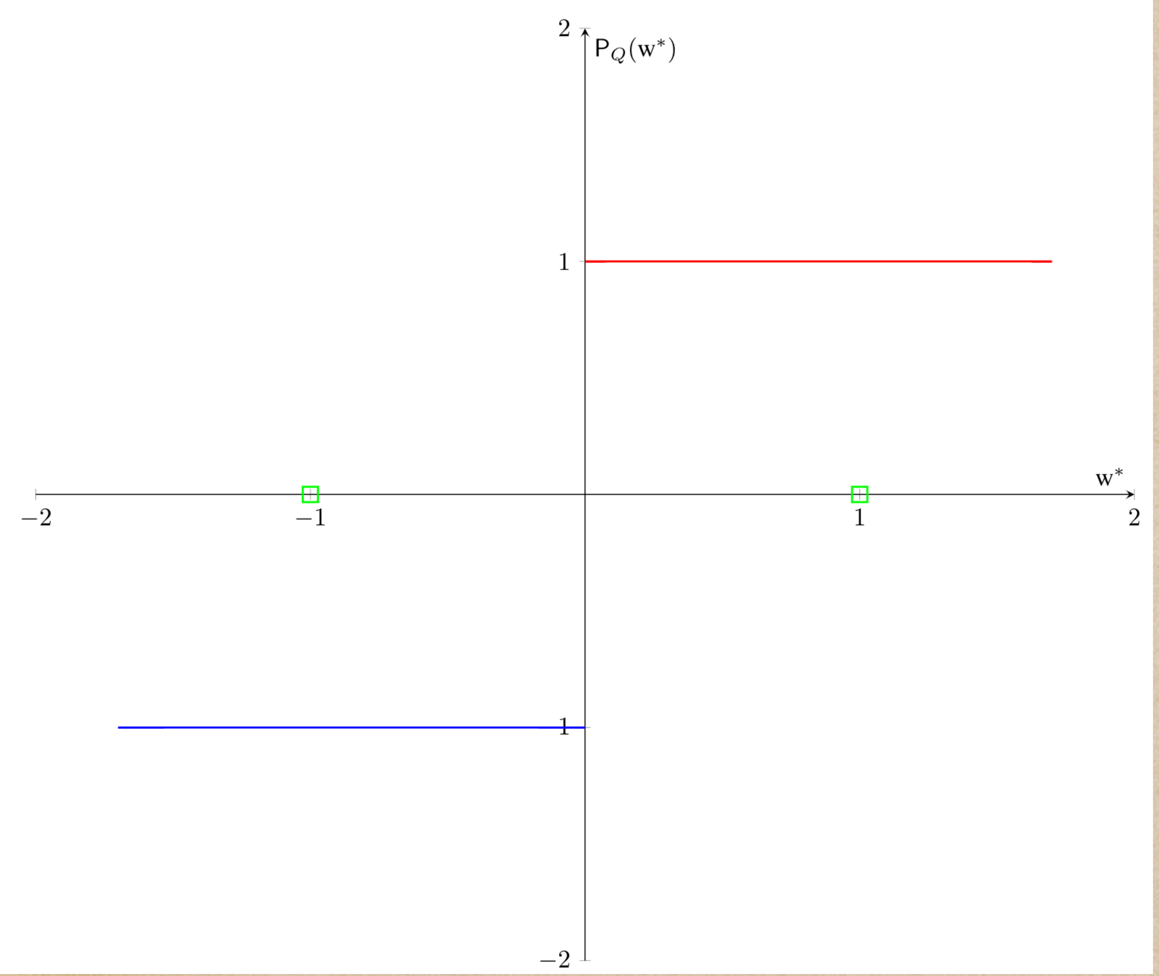

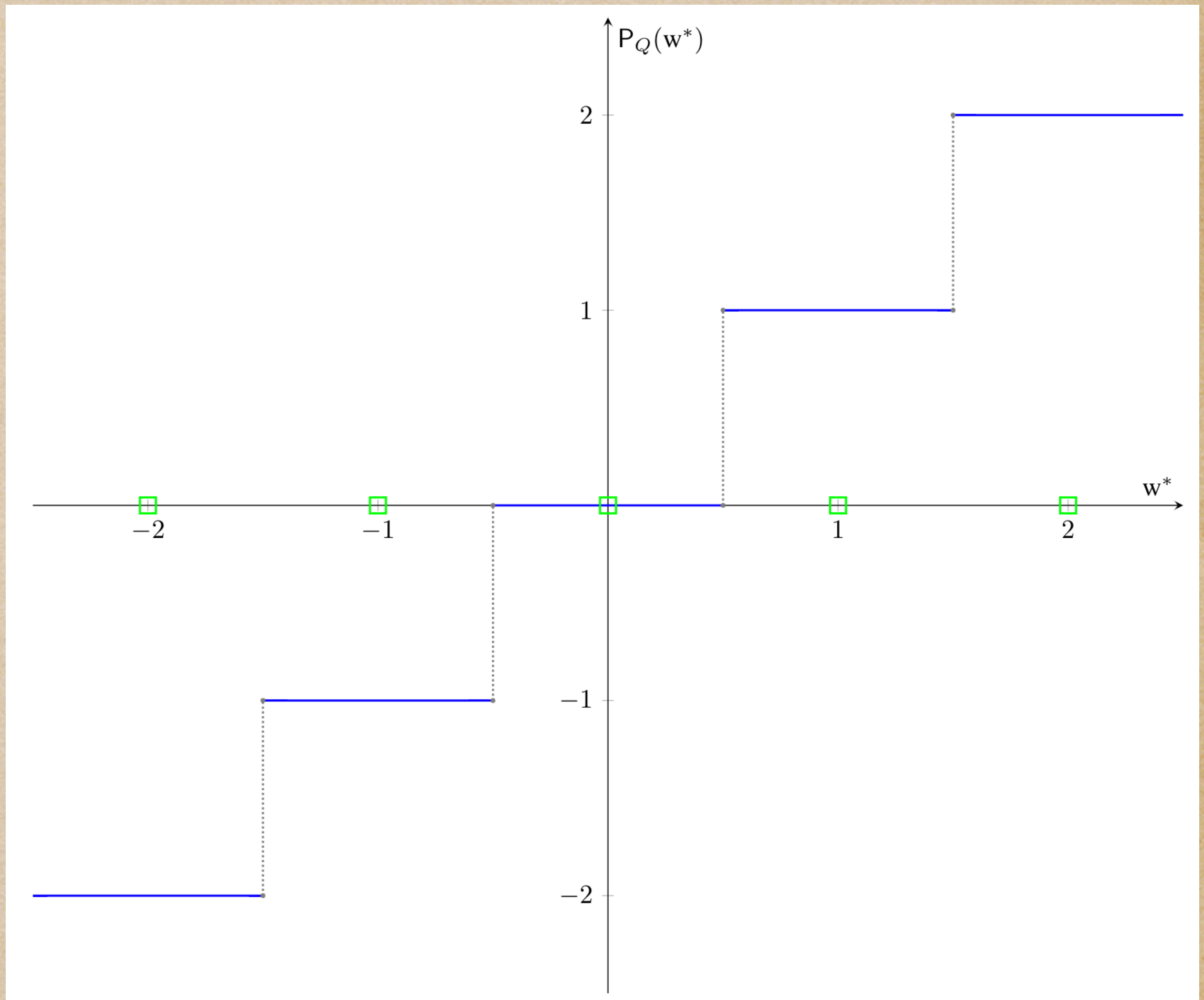

“Straight-through”

$$\min_{\mathbf{w}\in Q} \ell(\mathbf{w}) \xrightarrow{\text{reparameterize}} \min_{\mathbf{w}^*\in\mathbb{R}^d} \ell\left(\mathbf{P}(\mathbf{w}^*)\right)$$

- Y. Bengio, N. Léonard, and A. Courville. *Estimating or Propagating Gradients Through Stochastic Neurons for Conditional Computation*. arXiv (2013).

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$$\mathbf{w}^*_{t+1} = \mathbf{w}^*_t - \eta_t \, \cancel{\nabla \mathbf{P}(\mathbf{w}^*_t)} \cdot \widetilde{\nabla} \ell\big(\mathbf{P}(\mathbf{w}^*_t)\big)$$

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- Black magic is necessary? ∇P does not exist!

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Proximal Quantization

$$\mathbf{w}_{t+1} = \mathbf{P}\left(\mathbf{w}_t - \eta_t \widetilde{\nabla}\, \ell(\mathbf{w}_t)\right)$$

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- In implementation: $\eta_t \to \infty$!

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The Similarity and Subtlety

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- BC: $\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \;\; \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t)$
- PQ: $\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \;\; \mathbf{w}_{t+1}^* = \mathbf{w}_t - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t)$
- PT: $\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \;\; \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t^*)$

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- rBC: $\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t^*)$
- PT: $\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t^*)$

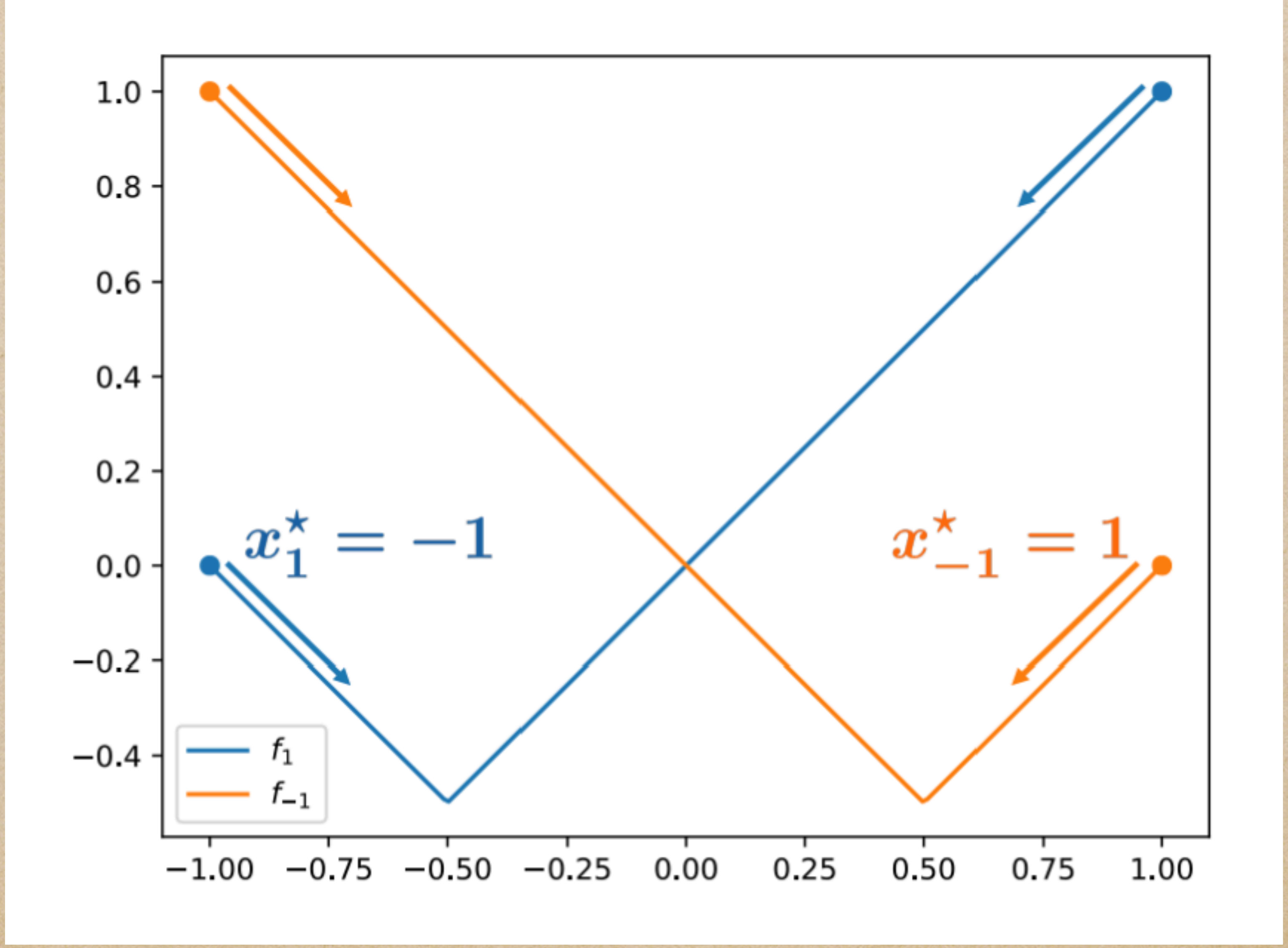
PT Doesn't Work At ALL

Model	Dataset	Full Precision	Binary		Ternary		Quaternary	
			without cal	with cal	without cal	with cal	without cal	with cal
ResNet20	CIFAR-10	92.01	10.17	17.71	10.00	11.89	9.30	35.41
ResNet56	CIFAR-10	93.01	10.41	39.15	10.00	9.99	10.06	58.31

- Cal: Training the BatchNorm layers for 1 epoch

The Problem of BC

$$\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t)$$



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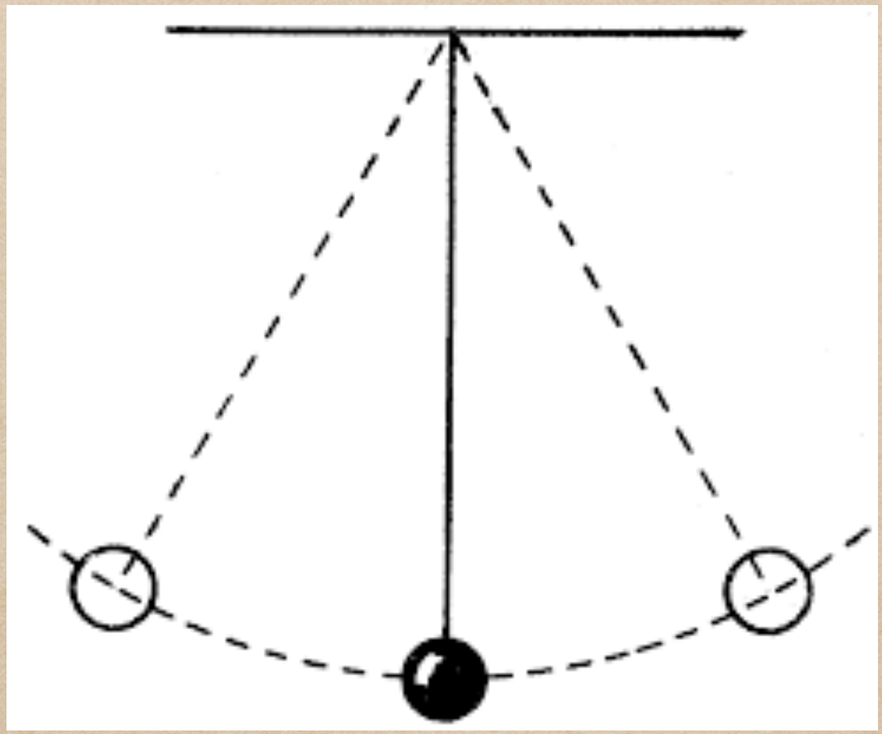
The Problem of PQ

$$\mathbf{w}_t = \mathrm{P}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t)$$



- Small update $\eta_t \widetilde{\nabla} \ell(\mathbf{w}_t)$ leads to $\mathbf{w}_{t+1} = \mathbf{w}_t$

What Makes a Good Quantizer?

Proximal Quantizer

$$\mathrm{P}_{\mathrm{r}}^{\nu}(\mathbf{w}^*) := \operatorname*{argmin}_{\mathbf{w}} \frac{1}{2\nu} \|\mathbf{w}^* - \mathbf{w}\|_2^2 + \mathrm{r}(\mathbf{w})$$

- If $\mathrm{r}(\mathbf{w}) = \iota_Q(\mathbf{w})$, reduce to projection: finding the discrete weight $\mathbf{w}$ in Q that is closest to the continuous weight $\mathbf{w}^*$.
- More generally, the regularizer $\mathrm{r}(\mathbf{w})$ penalizes deviation from discreteness (i.e. from Q).
- ν: allowing gradual transitioning from continuous to discrete.

How to choose the regularizer r?

- Typical choices: $\iota_Q(\mathbf{w})$, $\mathrm{dist}_Q(\mathbf{w})$, $\mathrm{dist}_Q^2(\mathbf{w})$
- From r to P: tedious and uninspiring calculation

$$\mathsf{P}_{\mathsf{r}}^{\nu}(\mathbf{w}^*) := \operatorname*{argmin}_{\mathbf{w}} \frac{1}{2\nu}\|\mathbf{w}^* - \mathbf{w}\|_2^2 + \mathsf{r}(\mathbf{w})$$

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- Don't!

A Direct Design Approach

Theorem 3.1 ([41, Proposition 3]). *A (possibly multi-valued) map* $\mathsf{P} : \mathbb{R} \rightrightarrows \mathbb{R}$ *is a proximal map (of some function* r*) if and only if it is (nonempty) compact-valued, monotone and has a closed graph. The underlying function* r *is unique (up to addition of constants) iff* P *is convex-valued, while* r *is convex iff* P *is nonexpansive (i.e.* 1*-Lipschitz continuous).*

Theorem 3.2. *Let* $\mathsf{P}_i : \mathbb{R}^d \rightrightarrows \mathbb{R}^d, i = 1, \ldots, k$ *be proximal maps. Then, the averaged map*

$$\mathsf{P} := \sum_{i=1}^k \alpha_i \mathsf{P}_i, \qquad \text{where } \alpha_i \geq 0, \ \ \sum_{i=1}^k \alpha_i = 1, \tag{9}$$

is also a proximal map. Similarly, the product map

$$\mathsf{P} := \mathsf{P}_1 \times \mathsf{P}_2 \times \cdots \times \mathsf{P}_k, \ \ \mathbf{w}^* = (\mathbf{w}_1^*, \ldots, \mathbf{w}_k^*) \mapsto \big(\mathsf{P}_1(\mathbf{w}_1^*), \ldots, \mathsf{P}_k(\mathbf{w}_k^*)\big) \tag{10}$$

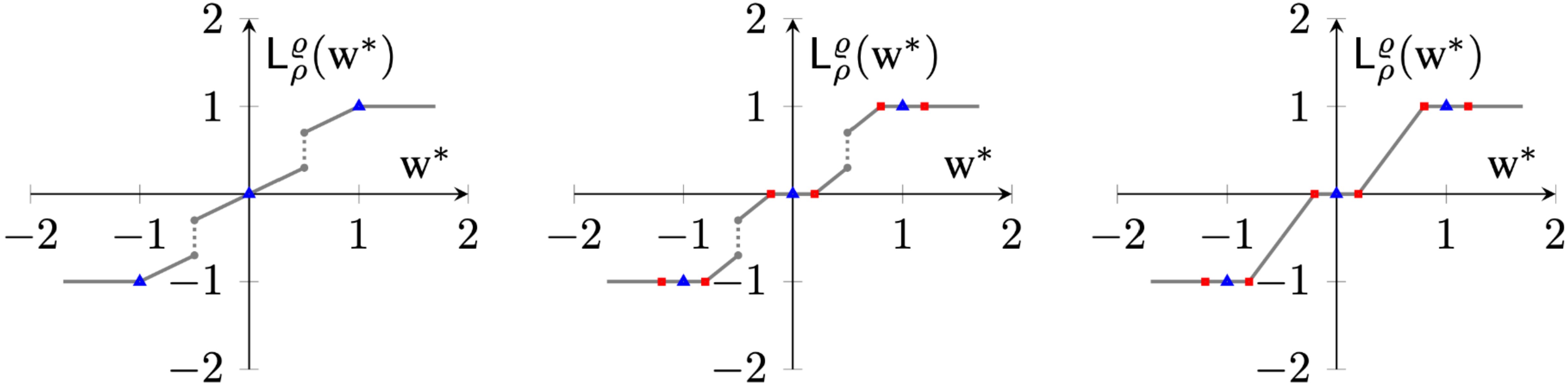
is a proximal map (from $\mathbb{R}^{dk}$ *to* $\mathbb{R}^{dk}$*).*

- Operationally, all we need to know is P.
- r is required for theoretical analysis: existence suffices

- Y. Yu, X. Zheng, M. Bowick and E. Xing. *Minimizing nonconvex and non-separable functions. AISTATS* (2015).

Example

$$\rho = \varrho = \frac{\nu}{2(\nu+1)}$$



(a) $\rho = 0, \varrho = 0.2$. (b) $\rho = \varrho = 0.2$. (c) $\rho = 0.2, \varrho = 0$.

Figure 1: Different instantiations of the proximal map L_ρ^ϱ in (13) for $Q = \{-1, 0, 1\}$.

- $\rho, \varrho \to 0$: P —> id, reduce to standard SGD
- $\rho, \varrho \to \infty$: P —> projection to Q, reduce to BC

Demystifying BC with 3 technical tools

Generalized Conditional Gradient

$$\min_{\mathbf{w}^*} f(\mathbf{w}^*) + g(\mathbf{w}^*)$$

- Y. Yu, X. Zhang and D. Schuurmans. *Generalized Conditional Gradient for Structured Sparse Estimation. JMLR* (2017).
- K. Bredies and D. Lorenz. *Iterated Hard Shrinkage for Minimization Problems with Sparsity Constraints*. SIAM SC (2008).

Generalized Conditional Gradient

$$\min_{\mathbf{w}^*} f(\mathbf{w}^*) + g(\mathbf{w}^*)$$

- Step 1: linearize f at current iterate $\mathbf{w}_t^*$

$$\min_{\mathbf{w}^*} f(\mathbf{w}_t^*) + (\mathbf{w}^* - \mathbf{w}_t^*) \cdot \nabla f(\mathbf{w}_t^*) + g(\mathbf{w}^*)$$

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- Step 2: solve above to obtain $\mathbf{z}_t^* = \nabla g^*\big(-\nabla f(\mathbf{w}_t^*)\big)$

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- Step 2: solve above to obtain $\mathbf{z}_t^* = \nabla g^*\big(-\nabla f(\mathbf{w}_t^*)\big)$
- Step 3: $\mathbf{w}_{t+1}^* = (1 - \lambda_t)\mathbf{w}_t^* + \lambda_t \mathbf{z}_t^*, \quad \lambda_t \in [0,1]$

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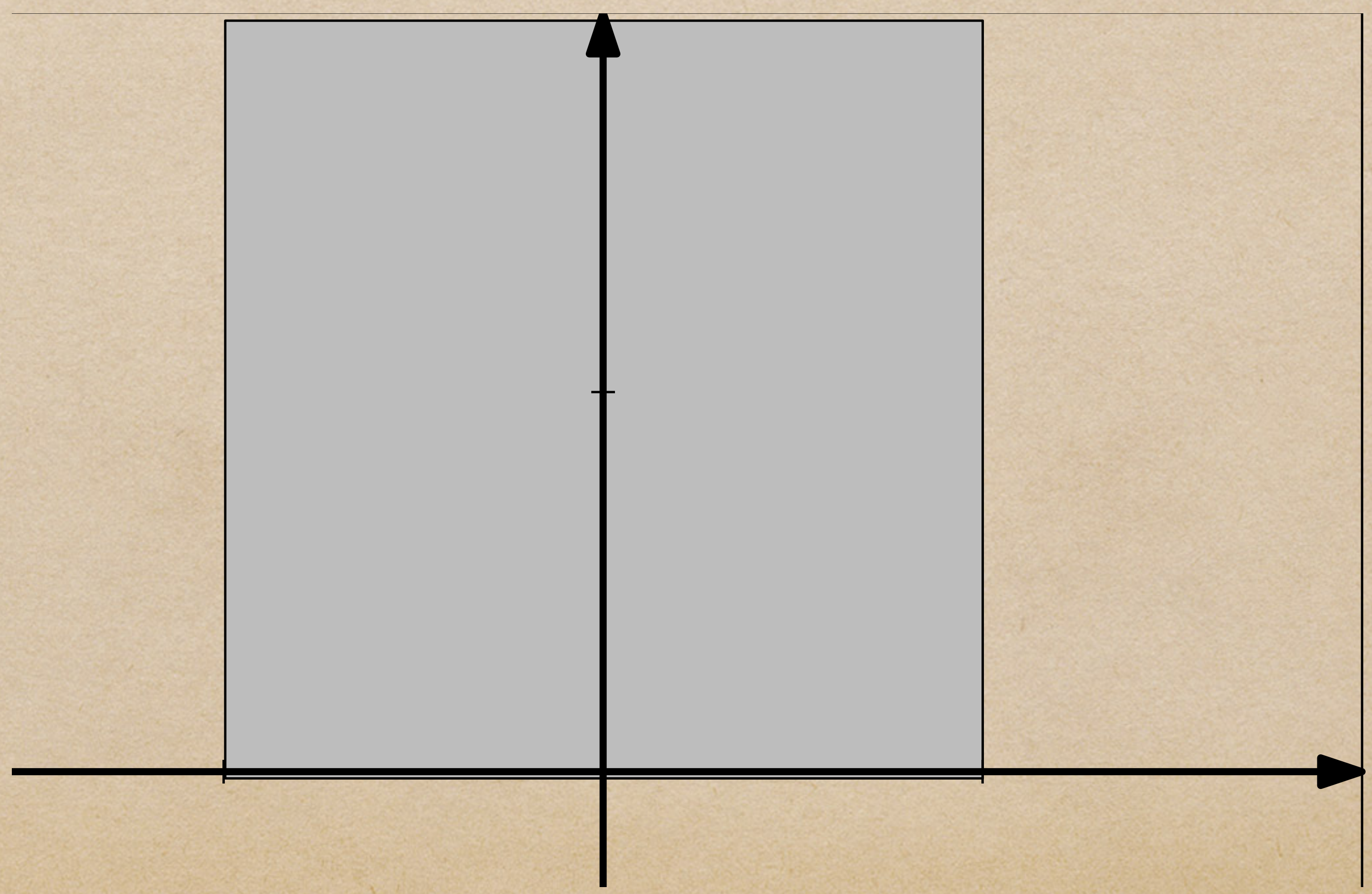
Example

$$\min_{|a|\leq 1, 0\leq b\leq 2} a^2 + (b+1)^2$$

g f



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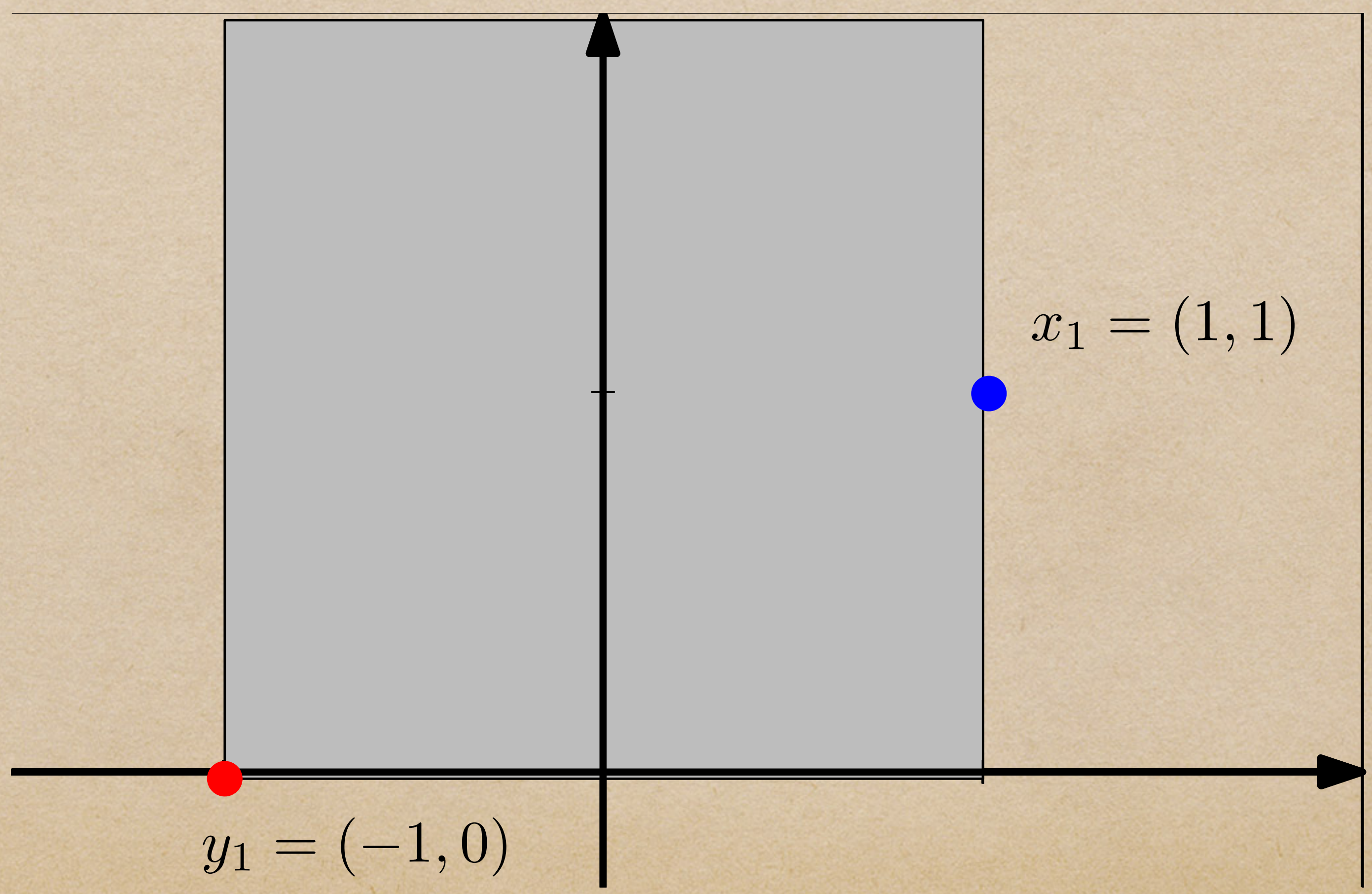

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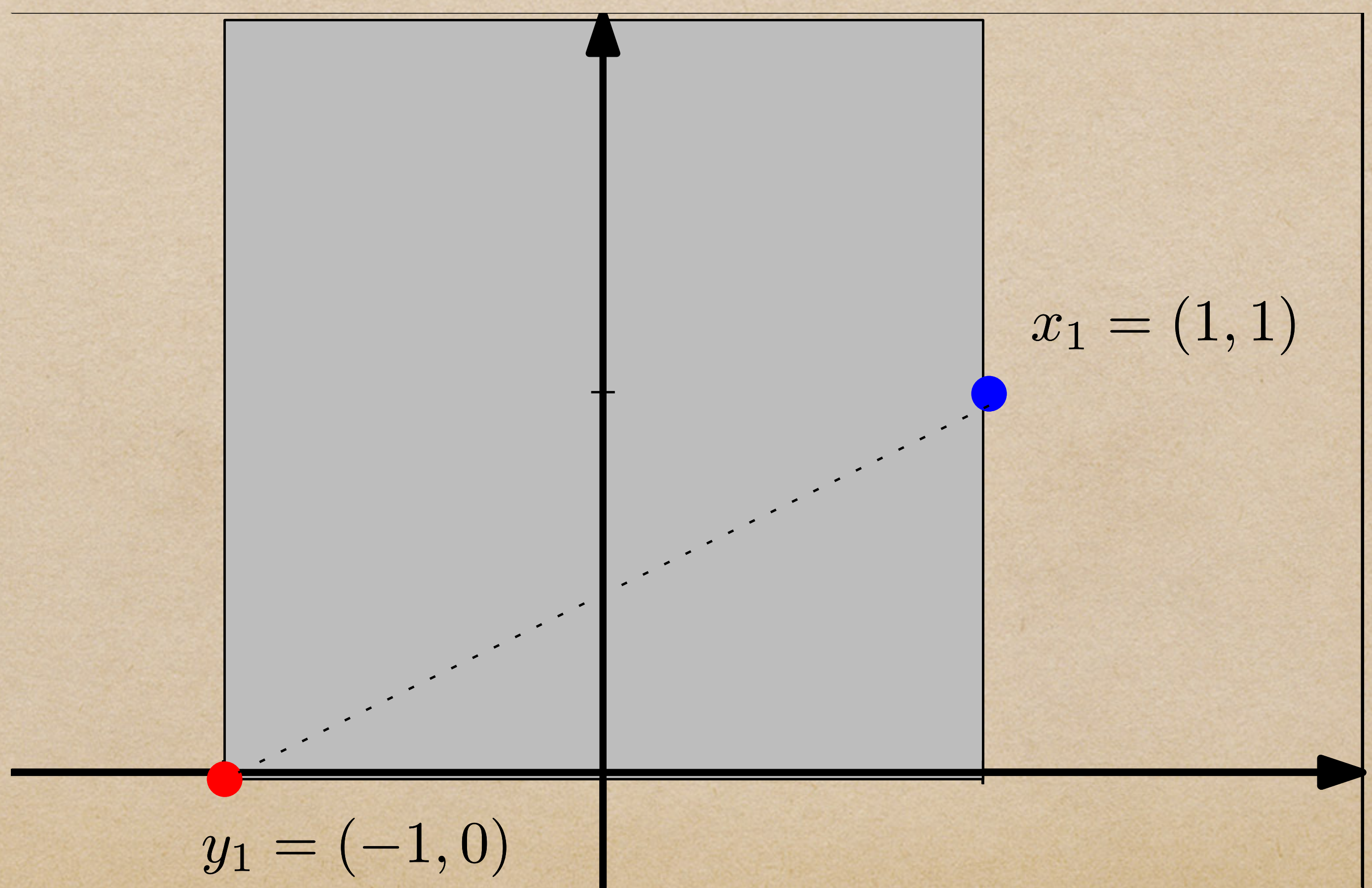

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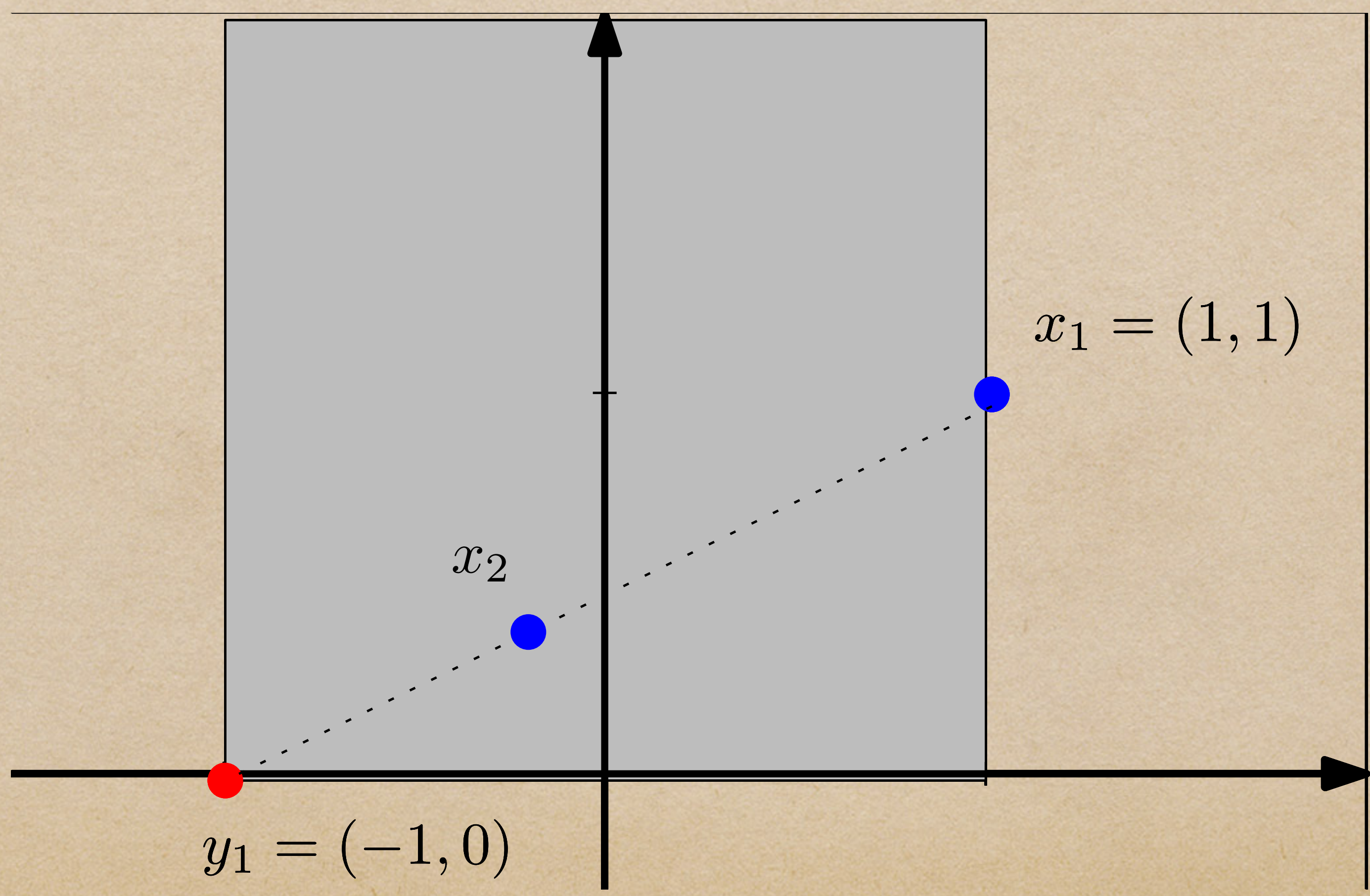

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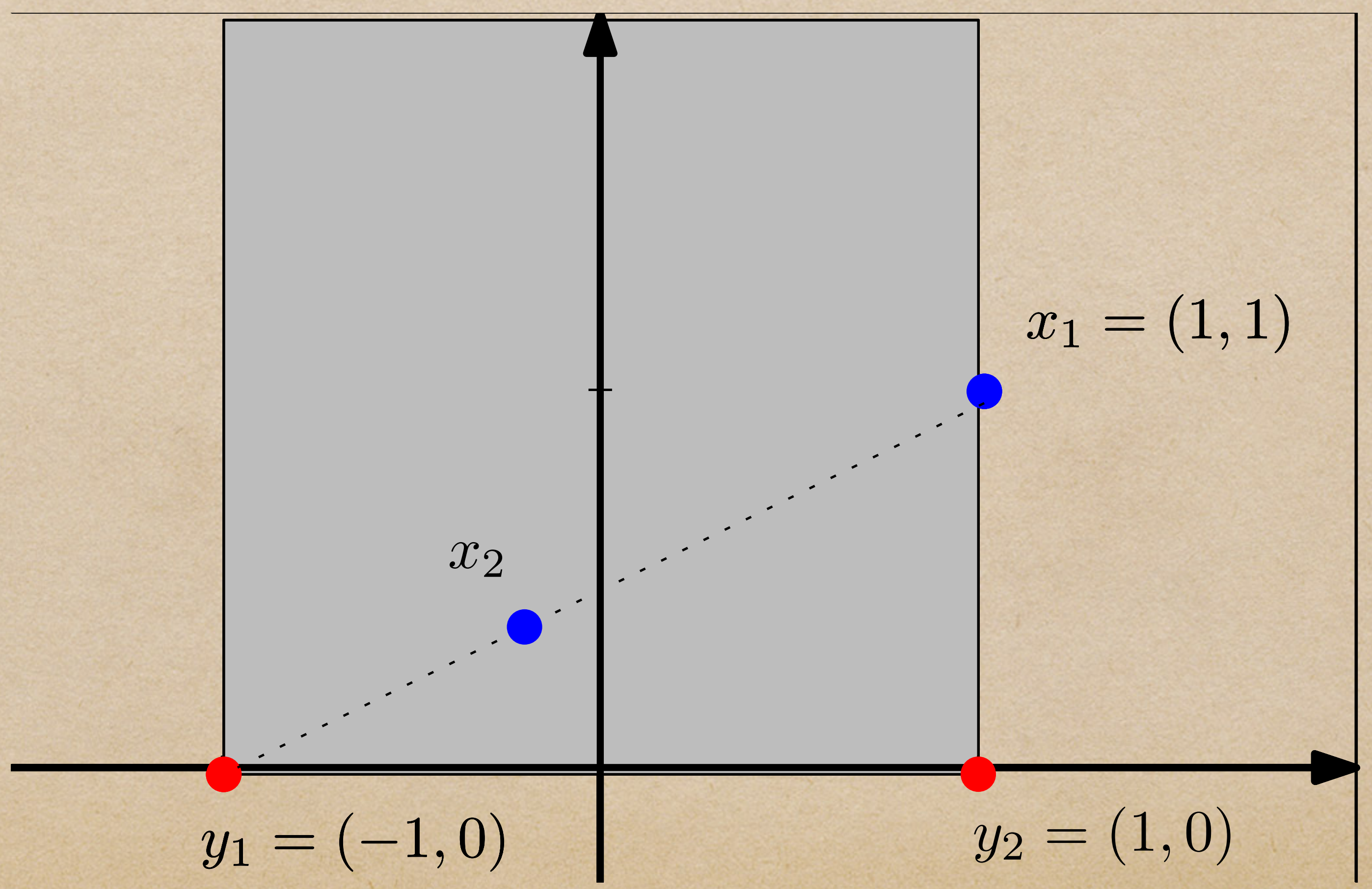

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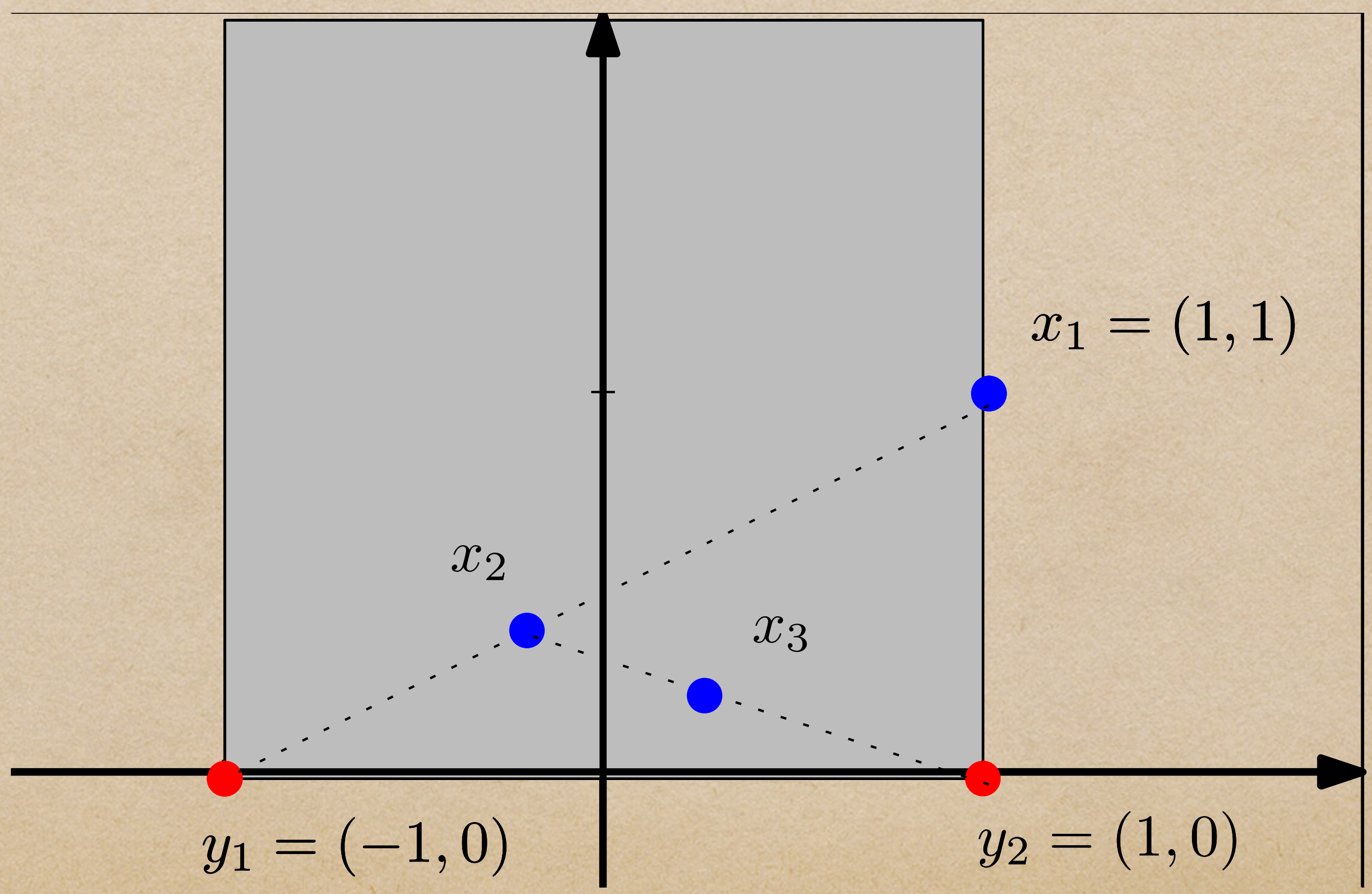

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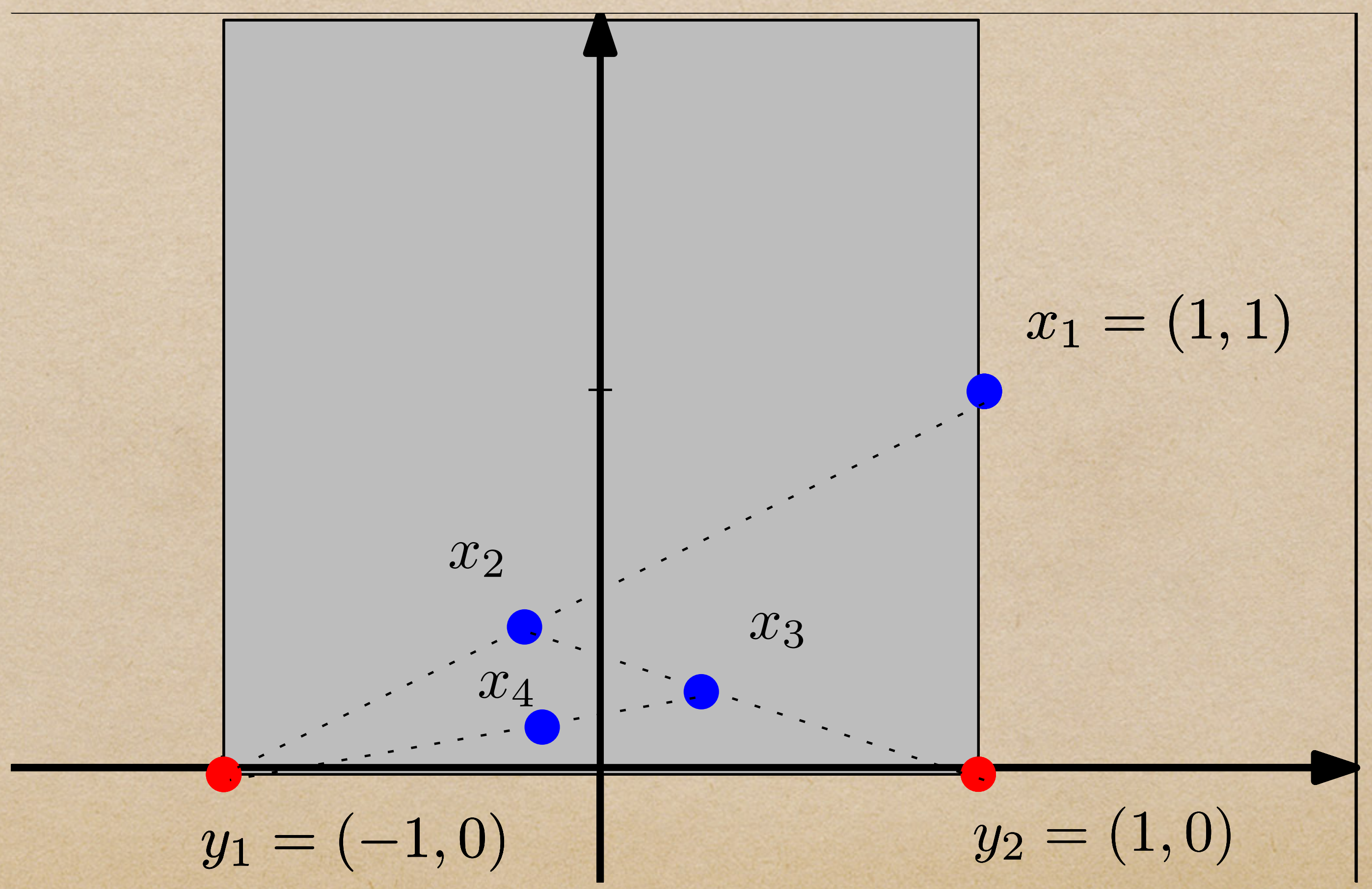


Primal and Dual

$$\min_{\mathbf{w}} \; \ell(\mathbf{w}) + \mathrm{r}(\mathbf{w}) \quad \Longleftrightarrow \quad \min_{\mathbf{w}^*} \; \ell^*(-\mathbf{w}^*) + \mathrm{r}^*(\mathbf{w}^*)$$

- Fenchel conjugate: $\ell^*(\mathbf{w}^*) := \max_{\mathbf{w}} \; \mathbf{w}^\top \mathbf{w}^* - \ell(\mathbf{w})$
- Twins: solving one helps solving the other.
- Example: linear programming duality.
- Note: The dual is always a convex problem.

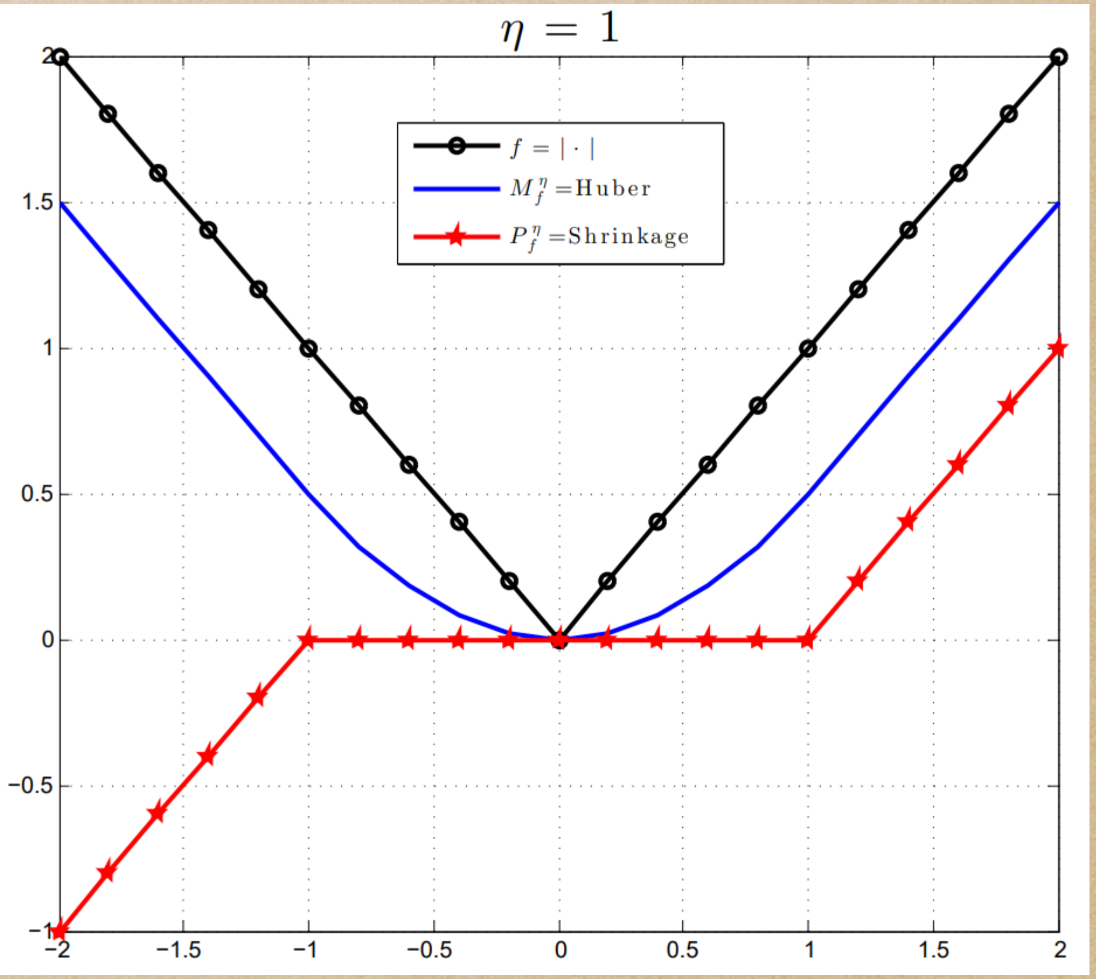
Moreau Envelope

$$\mathrm{M}_{\mathrm{r}}^{\mu}(\mathbf{w}^*) := \min_{\mathbf{w}} \frac{1}{2\mu}\|\mathbf{w}^* - \mathbf{w}\|_2^2 + \mathrm{r}(\mathbf{w})$$

- Smooth approximation
- μ controls error:

$$\mathrm{M}_{\mathrm{r}}^{\mu} \to \mathrm{r} \ \text{ as } \ \mu \downarrow 0$$

$$\nabla \mathrm{M}_{\mathrm{r}}^{\mu} = \frac{\mathrm{id} - \mathrm{P}_{\mathrm{r}}^{\mu}}{\mu} = \mathrm{P}_{\mathrm{r}^*}^{1/\mu}(\,\cdot\,/\mu)$$



Jean J. Moreau. *Fonctions convexes duales et points proximaux dans un espace hilbertien.* C.R.A.S. (1962)
Peter J. Huber. *Robust estimation of a location parameter.* Annals of Statistics (1964).

GCG —> Smoothened Dual

$$\min_{\mathbf{w}} \ \ell(\mathbf{w}) + \mathrm{r}(\mathbf{w})$$

$$\Downarrow$$

$$\min_{\mathbf{w}^*} \ \ell^*(-\mathbf{w}^*) + \mathrm{r}^*(\mathbf{w}^*)$$

$$\Downarrow$$

$$\min_{\mathbf{w}^*} \ \ell^*(-\mathbf{w}^*) + \mathrm{M}^{\mu}_{\mathrm{r}^*}(\mathbf{w}^*)$$

- As $\mu \downarrow 0$ we approach the original dual.
- $\mathrm{M}^{\mu}_{\mathrm{r}^*}$ is differentiable, with Lipschitz cont grad.

GCG —> Smoothened Dual

$$\min_{\mathbf{w}} \ \ell(\mathbf{w}) + r(\mathbf{w})$$

↓

$$\min_{\mathbf{w}^*} \ \ell^*(-\mathbf{w}^*) + r^*(\mathbf{w}^*)$$

↓

g

$$\min_{\mathbf{w}^*} \ \boxed{\ell^*(-\mathbf{w}^*)} + M^{\mu}_{r^*}(\mathbf{w}^*)$$

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GCG —> Smoothened Dual

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$$\min_{\mathbf{w}^*} \ \ell^*(-\mathbf{w}^*) + \mathrm{r}^*(\mathbf{w}^*)$$

g

$$\min_{\mathbf{w}^*} \ \ell^*(-\mathbf{w}^*) + \mathrm{M}^{\mu}_{\mathrm{r}^*}(\mathbf{w}^*)$$

f

- As $\mu \downarrow 0$ we approach the original dual.
- $\mathrm{M}^{\mu}_{\mathrm{r}^*}$ is differentiable, with Lipschitz cont grad.

Unpacking

- Step 1: $\mathbf{w}_t := \nabla \mathrm{M}^{\mu}_{\mathrm{r}*}(\mathbf{w}^*_t) = \mathrm{P}^{1/\mu}_{\mathrm{r}**}(\mathbf{w}^*_t/\mu)$
- Step 2: $\mathbf{z}^*_t := -\nabla \ell^{**}(\mathbf{w}_t)$
- Step 3: $\mathbf{w}^*_{t+1} = (1-\lambda_t)\mathbf{w}^*_t + \lambda_t \mathbf{z}^*_t, \quad \lambda_t \in [0,1]$
- f^{**} is the convex hull of f, the best convex approximation in some sense

Simplifying

$$\pi_t = \prod_{s=1}^{t} (1 - \lambda_s)$$

$$\eta_t = \frac{\lambda_t}{\pi_t}$$

- Upon change-of-variable:

$$\mathbf{w}_t = \mathrm{P}^{1/\mu}_{\mathrm{r}^{**}}(\pi_{t-1}\mathbf{w}_t^*/\mu), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \nabla \ell^{**}(\mathbf{w}_t)$$

- Allow $\mu = \mu_t = \pi_{t-1}$ to adapt with iteration:

$$\mathbf{w}_t = \mathrm{P}^{1/\pi_{t-1}}_{\mathrm{r}^{**}}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \nabla \ell^{**}(\mathbf{w}_t)$$

- Replace with non convex originals:

$$\mathbf{w}_t = \mathrm{P}^{1/\pi_{t-1}}_{\mathrm{r}}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \nabla \ell(\mathbf{w}_t)$$

The Nut is Cracked

- If $\mathbf{r}(\mathbf{w}) = \iota_Q(\mathbf{w})$, reduce to projection

$$\mathbf{w}_t = \mathrm{P}_{\mathsf{r}}^{1/\tau_{t-1}}(\mathbf{w}_t^*), \quad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \nabla \ell(\mathbf{w}_t)$$

- This is exactly BinaryConnect!

- Even for convex ℓ and r, new interpretation of the (regularized) dual averaging algorithm.

- Y. Nesterov. *Primal-dual subgradient methods for convex problems*. MP (2009).
- L. Xiao. *Dual Averaging Methods for Regularized Stochastic Learning and Online Optimization*. JMLR (2010).

Proximal Connect

Proximal Connect

$$\mathbf{w}_t = \mathrm{P}_{\mathrm{r}}^{\nu_t}(\mathbf{w}_t^*) \qquad \mathbf{w}_{t+1}^* = \mathbf{w}_t^* - \eta_t \widetilde{\nabla} \ell(\mathbf{w}_t)$$

$$\nu_t = 1 + \sum_{\tau=1}^{t-1} \eta_\tau$$

- $\nu_t = 1/\mu_t \to \infty$ and $\mathrm{P}_{\mathrm{r}}^{\nu_t} \to \mathrm{P}_Q$.
- Diverging ν_t was a crucial hack prior to our justification here.
- Easily derive improved convergence guarantees.

Experiments

Fine-tuning

Table 2: Fine-tuning pretrained ResNets. Final test accuracy: mean and std over three runs.

Model	Quantization	BC [10]	PQ [5]	rPC (ours)	PC (ours)
ResNet20	Binary	**90.31** (0.00)	89.94 (0.10)	89.98 (0.17)	**90.31** (0.21)
	Ternary	74.95 (0.16)	91.46 (0.06)	**91.47** (0.19)	91.37 (0.18)
	Quaternary	91.43 (0.07)	91.43 (0.21)	91.43 (0.06)	**91.81** (0.14)
ResNet56	Binary	92.22 (0.12)	92.33 (0.06)	92.47 (0.29)	**92.65** (0.16)
	Ternary	74.68 (1.4)	93.07 (0.02)	92.84 (0.11)	**93.25** (0.12)
	Quaternary	93.20 (0.06)	92.82 (0.16)	92.91 (0.26)	**93.42** (0.12)

End-to-end Training

Table 3: End-to-end training of ResNets. Final test accuracy: mean and std over three runs.

Model	Quantization	BC [10]	PQ [5]	rPC (ours)	PC (ours)
ResNet20	Binary	87.51 (0.21)	81.59 (0.75)	81.82 (0.32)	**89.92** (0.65)
	Ternary	27.10 (0.21)	47.98 (1.30)	47.17 (1.94)	**84.09** (0.16)
	Quaternary	89.91 (0.09)	85.29 (0.09)	85.05 (0.27)	**90.17** (0.14)
ResNet56	Binary	89.79 (0.45)	86.13 (1.71)	86.25 (1.50)	**91.26** (0.59)
	Ternary	30.31 (7.79)	50.54 (3.68)	42.95 (1.57)	**84.36** (0.75)
	Quaternary	90.69 (0.57)	87.81 (1.60)	87.30 (1.02)	**91.70** (0.14)

Conclusion

Summary

- Existing quantization algorithms are not that different from each other
- A convenient design of proximal quantizers
- BC is GCG applied to the smoothened dual
- ProxConnect unifies and extends SOTA
- Open possibilities for acceleration and new applications

Thank you!

[redacted]