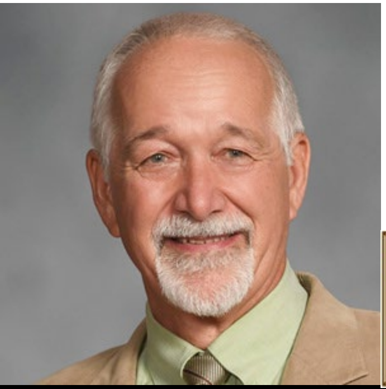
Quantum Interior Point Methods (QIPMs) with Iterative Refinement for Linear and Semidefinite Optimization

Tamás Terlaky

Alcoa Foundation Endowed Chair Professor
Quantum Computing Optimization Lab (QCOL)
Dept. of Industrial and Systems Engineering, Lehigh University



Quantum Computing Optimization Lab QCOL Lehigh University

ISE Industrial and Systems Engineering

LEHIGH UNIVERSITY

Quantum Computers are here

& we need more optimizers & OR in

Quantum Computing

UN: IYQST -- INFORMS: QCOR

The United Nation declared 2025
as the International Year of
Quantum Science and Technology

Quantum Computing (QC) is Here!

NISQ

Is QC really computing?

Hybrid classic-QC

Why is it important?

Why QC optimization?

Why to get involved?

NISQ devices v/s quantum annealers

Pro's and Con's

Opportunities & challenges

Quantum supremacy

Quantum advantage

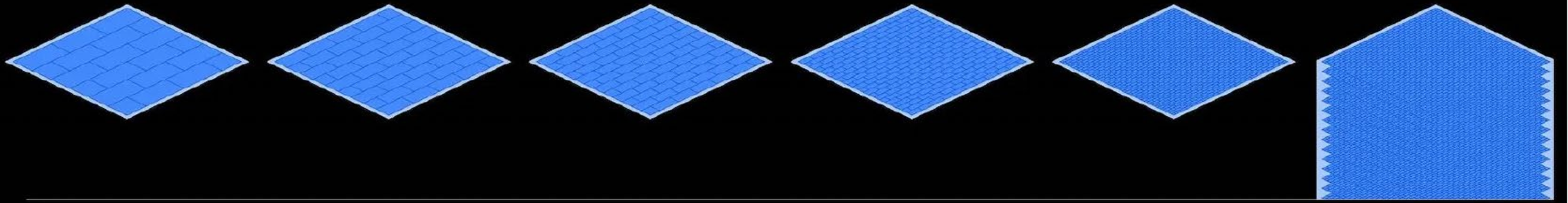
QC Capacity Grows Exponentially-IBM

Scaling IBM Quantum technology

Superconducting qubits

IBM

IBM Q System One (Released)		(In development)		Next family of IBM Quantum systems	
2019	2020	2021	2022	2023	and beyond
27 qubits *Falcon*	65 qubits *Hummingbird*	127 qubits *Eagle*	433 qubits *Osprey*	1,121 qubits *Condor*	Path to 1 million qubits and beyond *Large scale systems*
Key advancement Optimized lattice	Key advancement Scalable readout	Key advancement Novel packaging and controls	Key advancement Miniaturization of components	Key advancement Integration	Key advancement Build new infrastructure, quantum error correction



Quantum Computing Challenges

We are having **NISQ**

(**N**oisy **I**ntermediate-**S**cale **Q**uantum) devices

QC challenges

The promise:

Fault Tolerant Logical Qubits

Are coming!

(Not only on QC simulators)

Conic Linear Optimization

Primal-dual pair of CLO problems is given as

$$(P)\quad \min \quad c^T x \qquad\qquad (D)\quad \max \quad b^T y$$

$$\text{s.t.}\quad Ax - b \in \mathcal{C}_1 \qquad\qquad \text{s.t.}\quad c - A^T y \in \mathcal{C}_2^*$$

$$x \in \mathcal{C}_2 \qquad\qquad y \in \mathcal{C}_1^*,$$

where $b, y \in \mathbb{R}^m$, $c, x \in \mathbb{R}^n$, $A : m \times n$ matrix, $\mathcal{C}_1, \mathcal{C}_2$ are convex cones and $\mathcal{C}_i^* = \{s \in \mathbb{R}^n : x^T s \geq 0, \forall x \in \mathcal{C}_i\}$ are the dual cones for $i = 1, 2$.

These are solvable efficiently (in polynomial time) by using Interior Point Methods. LO is based on polyhedral cones.

Be careful! Perfect duality, strict complementarity lost.

Are all convex cones good???

NOT

IPMs~70-40: Why IPMs "failed" by ~1970

Computers ~1970

IBM360: top computer that time

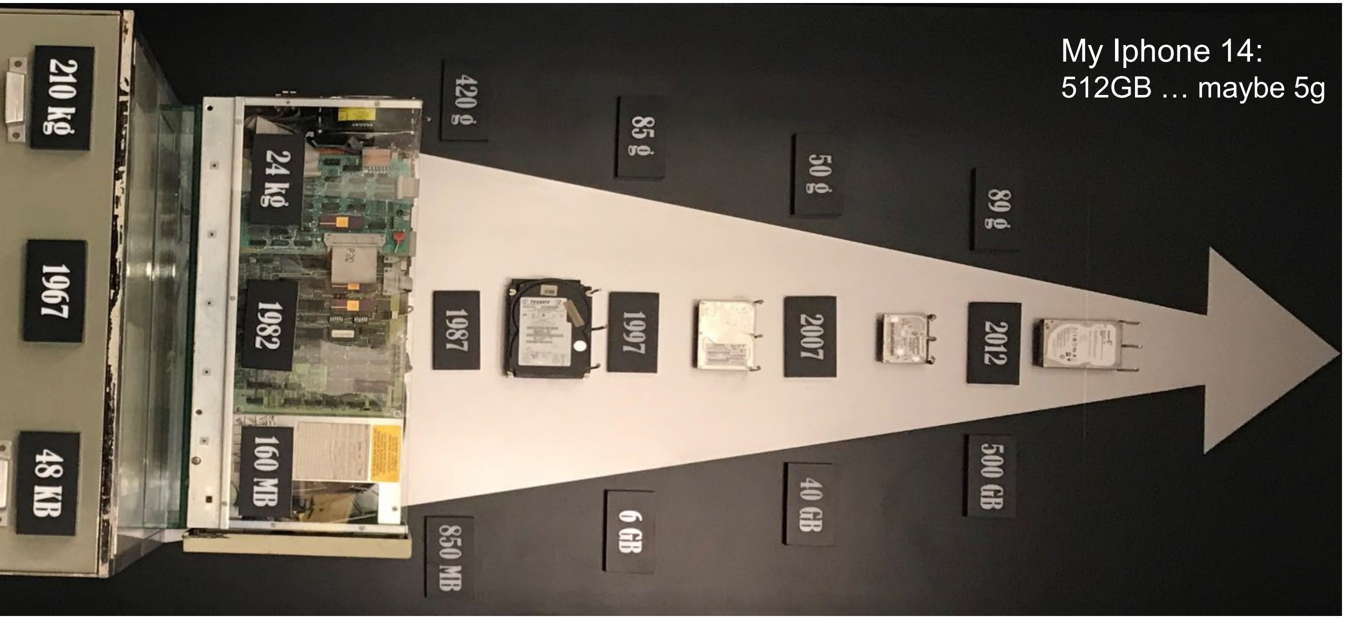
- Gym size room, Operators
- Memory: 128-256 KB
- Hard disk: 7.2MB-400MB
- Large Storage: tape drives
- Input: punch cards

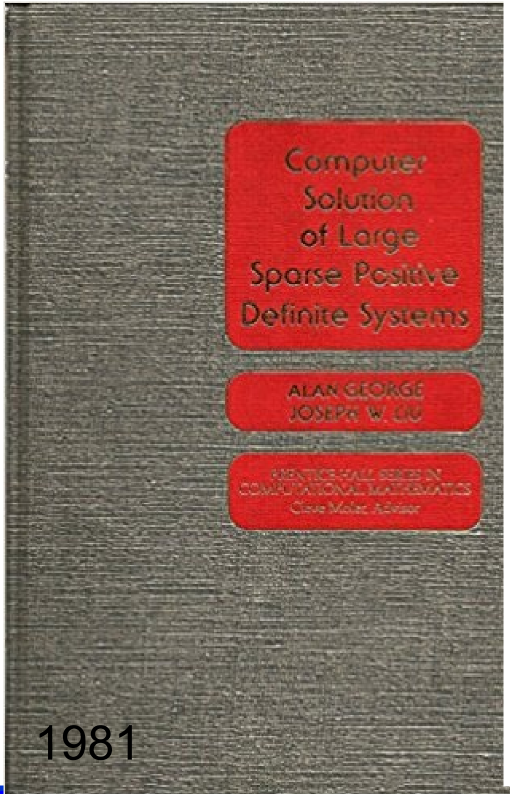
NO:
- Double precision arithmetic
- Regularization
- Sparse matrix methods
- Automatic differentiation

40: What made the IPM revolution possible?

- Plenty of memory
- Complexity Theory ⟹ Control of Algorithms
- <u>Polynomial time algorithm – Ellipsoid Method</u>
- Mainframe ⟹ Workstations ⟹ Desktop PC
- Sparse matrix theory and packages
- Automatic Differentiation

<u>All stars lined up JIT for the "IPM Revolution"</u>





The New York Times

NEW YORK, WEDNESDAY, NOVEMBER 7, 1979

A Soviet Discovery Rocks World of Mathematics

Russian's Surprise Problem-Solving Discovery Reported

Quantum Interior Point Methods

We also have Quantum Central Path Methods …

Thanks to joint work with the QCOL team:
Brandon Augustino, ***Mohammadhossein Mohamadisiahroudi***, Ramin Fakhimi,
Arielle Carr, Pouya Sampourmahani, Zeguan Wu, Luis Zuluaga
Giacomo Nannicini (USC), Xiaodi Wu, Jiaqi Leng (UMd)

Quantum Interior Point Methods (QIPMs)

- QIPMs are Hybrid Classic-Quantum algorithms
- For solving real world optimization problems

Needed:

Efficient, reliable

Quantum Computers & Quantum Linear Algebra

Data flow in hybrid classic-quantum algorithms:

- Classic → QRAM → QSolve → QTomography → Classic solution

Quantum Linear Algebra

Need to solve Newton svstems – “accurately, fast”

$$Mz = \sigma, \; M|z\rangle = |\sigma\rangle$$

Algorithm	Complexity
Factorization (e.g. Cholesky)	$\mathcal{O}(p^3)$
Conjugate Gradient	$\mathcal{O}(pd\sqrt{\kappa}\log(\frac{1}{\epsilon}))$
HHL + QTA	$\mathcal{O}(\text{polylog}(p)\frac{d^2\kappa^2\|\sigma\|}{\|M\|\epsilon}) \times \mathcal{O}(\frac{p\|\sigma\|}{\|M\|\epsilon})$
VTAA-HHL + QTA	$\mathcal{O}(\text{polylog}(p)\frac{d^2\kappa\|\sigma\|}{\|M\|\epsilon}) \times \mathcal{O}(\frac{p\|\sigma\|}{\|M\|\epsilon})$
QLSA (Wossnig, et al. 2018) + QTA	$\mathcal{O}(\text{polylog}(p)\frac{\kappa\|\sigma\|}{\|M\|\epsilon}) \times \mathcal{O}(\frac{p\|\sigma\|}{\|M\|\epsilon})$
QLSA (Childs, et al. 2017) + QTA	$\mathcal{O}(\text{polylog}(\frac{p\kappa\|\sigma\|}{\|M\|\epsilon})d\kappa) \times \mathcal{O}(\frac{p\|\sigma\|}{\|M\|\epsilon})$
QLSA (Carrera, et al. 2020) + QTA	$\mathcal{O}(\text{polylog}(\frac{p\kappa\|\sigma\|}{\|M\|\epsilon})d\kappa) \times \mathcal{O}(\frac{p\|\sigma\|}{\|M\|\epsilon})$
QLSA (Chakraborty, et al. 2018) + QTA	$\mathcal{O}(\text{polylog}(\frac{p\|\sigma\|}{\epsilon})\kappa\|M\|_F) \times \mathcal{O}(\frac{p\|\sigma\|}{\epsilon})$

with tomography

*κ is the condition number of M,
d is maximum number of non-zero elements in each row and column in M,
p is the number of rows/columns of M, and
ϵ is the error of Linear Equation Solver.

Novel IR-IF-QIPMs

QLSAs have **quantum advantage** w.r.t. the dimension, and
quantum disadvantage w.r.t. **condition number** & **accuracy** of solution

Tomography (extracting the solution) is costly

… direct use Leads to Inexact Infeasible QIPMs (II-QIPMs)

Best complexity QIPMs feature:

- **Novel Inexact Feasible IPMs (IF-QIPMs)**
- Error (inexactness) & condition number dependence need to be addressed
- **Solution: Iterative Refinement (IR) "inside and outside"**
  - **IR inside: for the Newton System**
  - **IR outside: at the problem level – limits condition number**
- **IR-IF-QIPMs: High precision solution through low precision computation.**

Semidefinite Optimization

Let

- $b \in \mathbb{R}^m$
- matrices $A_1, \ldots, A_m, C \in \mathcal{S}^n$

Then, the primal-dual Semidefinite Optimization (SDO) pair is given by:

$$z_P = \inf_X \{\operatorname{tr}(CX) : \operatorname{tr}(A_i X) = b_i, \ \forall i \in [m], X \succeq 0\}$$

$$z_D = \sup_{y,S} \left\{ b^\top y : \sum_{i=1}^m y_i A_i + S = C, \ S \succeq 0, y \in \mathbb{R}^m \right\}$$

where

- $[m] = \{1, \ldots, m\}$
- $S = C - \sum_{i \in [m]} y_i A_i \succeq 0$ is the slack matrix of the dual problem
- $\mathcal{S}^n$ is the cone of $n \times n$ symmetric matrices
- We assume that the matrices $A_1, \ldots, A_m$ are linearly independent

SDO

First SDO formed by

Bellman-Fang 1963

Craven-Mond 1981

General Theory of IPMs by Nesterov and Nemirovskii

Central Path, Newton System

For $\nu > 0$, assuming *interior point condition*, and linear independence of the matrices $A^{(i)}$, the central path is the solution set of equation system

$$\operatorname{tr}(A_i X) = b_i \ \forall i \in [m], \ X \succ 0$$

$$\sum_{i \in [m]} y_i A_i - S = C, \ S \succ 0$$

$$XS = \nu I,$$

- Linearizing the central path eqns gives the Newton linear system:

$$X \Delta S + \Delta X \ S = \sigma \nu I - XS$$

$$\Delta S \in L \quad \Delta X \in L^{\perp}.$$

No symmetric solution for this linear system.

Symmetrization needed!

First proposal for a QIPM

- Kerenidis and Prakash (2018) made the first effort at a quantum interior point method
- Use quantum random access memory (QRAM) and block encodings to solve the Newton linear system
- Small neighborhood IPM:

$$\mathcal{N}_F(\gamma) = \left\{(X, y, S) \in \mathcal{P}^0 \times \mathcal{D}^0 : \left\| X^{1/2} S X^{1/2} - \nu I \right\|_F \leq \gamma\nu \right\}$$

- They posit a worst cast running time of

$$\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\xi},\frac{1}{\epsilon}}\left(\frac{n^{2.5}}{\xi^2}\mu\kappa^3 \log\frac{1}{\epsilon}\right)$$

  for SDPs
  - The term $\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\xi}}\left(\frac{n^2\kappa^2}{\xi^2}\right)$ comes from a tomography subroutine
  - $\mu \leq n$ and κ are factors corresponding to the QLSA
- For solving LPs, the running time is

$$\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\xi},\frac{1}{\epsilon}}\left(\frac{n^{1.5}}{\xi^2}\mu\kappa^3 \log\frac{1}{\epsilon}\right)$$

No symmetrization.
Not taking account QC error
⟹ inexact, infeasible QIPM.
Condition number dependence.

Symmetrizing the Newton System

Symmetrization is a linear transformation:

$$H_P(M) = \frac{1}{2}\left[PMP^{-1} + P^{-T}M^TP^T\right].$$

for a given invertible matrix P

The Alizadeh-Haeberly-Overton (AHO) direction is given by

$$P = I$$

The Nesterov-Todd (NT) direction is given by

$$P = W^{-1/2}$$

where

$$\begin{aligned} W &= S^{-1/2}(S^{1/2}XS^{1/2})^{1/2}S^{-1/2} \\ &= X^{-1/2}(X^{1/2}SX^{1/2})^{1/2}X^{-1/2} \end{aligned}$$

Additionally there is the so called HKM direction for which

$$P = S^{1/2}$$

Quantum Tomography introduces error.

Gives **Inexact Infeasible QIPMs (II-QIPM)**

$$\begin{pmatrix} 0 & \mathcal{A} & 0 \\ \mathcal{A}^\top & 0 & \mathcal{I} \\ 0 & \mathcal{E} & \mathcal{F} \end{pmatrix} \begin{pmatrix} \Delta y \\ \Delta X \\ \Delta S \end{pmatrix} = \begin{pmatrix} \xi_p \\ \xi_d \\ \sigma\nu I - H_p(XS) + \xi_c \end{pmatrix}$$

II-IPMs have worse iteration complexity than Feasible IPMs!

We cannot avoid tomography thus:

Need to develop

Inexact Feasible IPMs and QIPMs

Inexact-Feasible QIPMs for SDO

Feasibility implies $\Delta X \in \mathrm{Null}(\mathcal{A})$ and $\Delta S \in \mathcal{R}(\mathcal{A})$

where $\mathrm{Null}(\mathcal{A}) \equiv$ nullspace of $\mathcal{A}_s$

$\mathcal{R}(\mathcal{A}) \equiv$ rowspace of $\mathcal{A}_s$

Let Q_2 be a basis of the null space of $\mathcal{A}_s$, then we set:

$$\textbf{svec}(\Delta X) = \textbf{svec}(Q_2 \Delta z)$$

$$\textbf{svec}(\Delta S) = \textbf{svec}(-\mathcal{A}_s^\top \Delta y)$$

So we get the new Newton system, called OSS (Orthogonal Subspace System)

$$\begin{bmatrix} \mathcal{E}Q_2 & \mathcal{F}(-\mathcal{A}_s^\top) \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta y \end{bmatrix} = \textbf{svec}(\sigma \nu I - H_p(XS)) \qquad \text{(OSS)}$$

Regardless of QLSA+QTA error, primal-dual feasibility is preserved!

Analysis of Feasible IPMs can be recovered!

Inexact-Feasible QIPM

Algorithm Inexact-Feasible Quantum Interior Point Method

Input: $\epsilon, \delta > 0$; $\sigma = 1 - \delta/\sqrt{n}$; $\beta, \gamma \in (0,1)$
Output: An ϵ-optimal primal dual pair (X, y, S)
Choose $(X^{(0)}, y^{(0)}, S^{(0)}) \in \mathcal{N}_F(\gamma)$
Set $\nu^{(0)} = \frac{X^{(0)} \bullet S^{(0)}}{n}$
Compute bases for $\mathcal{R}(\mathcal{A}_s) = \mathcal{A}_s^\top$ and $\text{Null}(\mathcal{A}_s) = Q_2$
while $\nu > \epsilon$ **do**

1. $\nu^{(k)} \leftarrow \frac{\text{tr}(X^{(k)} S^{(k)})}{n}$
2. Compute matrices $(X^{(k)})^{-1}$ and $(S^{(k)})^{-1}$, $P^{(k)}$ classically.
3. Using block-encodings, solve Newton system to construct inexact search direction $|\Delta z^{(k)} \circ \Delta y^{(k)}\rangle$. Quantum
4. Obtain classical estimate $\overline{\Delta z}^{(k)}, \overline{\Delta y}^{(k)}$ of $\Delta z^{(k)}, \Delta y^{(k)}$ using vector state tomography. Quantum
5. Use classical estimate $\overline{\Delta z}^{(k)}, \overline{\Delta y}^{(k)}$ to obtain classical estimate $\overline{\Delta X}^{(k)}, \overline{\Delta S}^{(k)}$ of $\Delta X^{(k)}, \Delta S^{(k)}$
6. Update current solution

$$X^{(k+1)} \leftarrow X^{(k)} + \overline{\Delta X}^{(k)}, \quad S^{(k+1)} \leftarrow S^{(k)} + \overline{\Delta S}^{(k)} \text{ and } y^{(k+1)} \leftarrow y^{(k)} + \overline{\Delta y}^{(k)}$$

$$k \leftarrow k + 1$$

end

Theorem:

Data stored in QRAM.
The IF-QIPM requites at most

$$\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\epsilon}}\left(n^{3.5}\frac{\kappa^2}{\epsilon}\right)$$

QRAM access and

$$\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\epsilon}}\left(n^{4.5}\right)$$

arithmetic operations.

Advantage w.r.t. the Dimension.
Disadvantage w.r.t. condition number and precision

Iterative refinement for LSP

Solve the linear system $Mv=w$

Algorithm Iterative Refinement for LSPs

Input: Error tolerances $0 < \zeta \ll \xi < 1$, bound on norm of solution θ

Output: A ζ-precise solution v to $\frac{M}{\theta}v = \frac{w}{\theta}$

Normalize the data $(\widetilde{M}, \tilde{w}) \leftarrow \theta^{-1}(M, w)$

Initialize: $x^{(0)} \leftarrow 0$, $r^{(0)} \leftarrow \tilde{w}$, $\eta^{(0)} \leftarrow 1$, $k \leftarrow 0$

while $\|r\| > \zeta$ **do**

1. $\bar{u}^{(k)} \leftarrow$ **solve** $(\widetilde{M}, \eta^{(k)} r^{(k)})$ using $O_{LS}(\xi)$ **Quantum-solve**
2. $\tilde{u}^{(k+1)} \leftarrow \frac{\|\eta^{(k)} r^{(k)}\|}{\|\widetilde{M}\bar{u}^{(k)}\|}\bar{u}^{(k)}$
3. Update solution: $v^{(k+1)} \leftarrow v^{(k)} + \frac{1}{\eta^{(k)}}\tilde{u}^{(k)}$
4. Update residual $r^{(k+1)} \leftarrow \tilde{w} - \widetilde{M}x^{(k+1)}$
5. Update scaling factor $\eta^{(k+1)} \leftarrow \|r^{(k+1)}\|^{-1}$
6. $k \leftarrow k + 1$

end

Here O_{LS} is defined as QLSA+QTA.

Theorem: For the iterates of the **IR Algorithm** we have

(a) $\eta^{(k)} \geq \frac{1}{\xi^k}$,

(b) $\left\|\tilde{w} - \widetilde{M}v^{(k)}\right\| \leq \xi^k$.

The **IR Algorithm** terminates in at most

$$\mathcal{O}\left(\log\left(\frac{1}{\zeta}\right)\right)$$

iterations.

IR for LSP: Complexity

Theorem: Let the problem data stored in QRAM, and use fixed precision oracle O_{LS} at each iteration. Then each iteration of the **IR Algorithm** requires at most

$$\mathcal{O}\left(d\kappa_M \cdot \text{polylog}\left(d, \kappa_M\right)\right)$$

QRAM access and $\mathcal{O}(ds)$ classic arithmetic operations.

Corollary: Let the problem data stored in QRAM, and use fixed precision oracle O_{LS} at each iteration. Then setting $\zeta = \frac{\epsilon}{\theta}$, the **IR Algorithm** obtains an ϵ-precise solution of the linear system $Mv=w$ with at most

$$\mathcal{O}\left(d\kappa_M \cdot \text{polylog}\left(d, \kappa_M, \theta, \epsilon^{-1}\right)\right)$$

QRAM access and $\mathcal{O}\left(ds \cdot \text{polylog}\left(d, \kappa_M, \theta, \epsilon^{-1}\right)\right)$ arithmetic operations.

IF-QIPM with Inner IR

Input: $\epsilon, \delta > 0$; $\sigma = 1 - \delta/\sqrt{n}$; $\beta, \gamma \in (0, 1)$

Output: An ϵ-optimal primal dual pair (X, y, S)

Choose $(X^{(0)}, y^{(0)}, S^{(0)}) \in \mathcal{N}_F(\gamma)$

Set $\nu^{(0)} = \frac{X^{(0)} \bullet S^{(0)}}{n}$

Compute bases for $\mathcal{R}(\mathcal{A}_s) = \mathcal{A}_s^\top$ and $\text{Null}(\mathcal{A}_s) = Q_2$

while $\nu > \epsilon$ **do**

1. $\nu^{(k)} \leftarrow \frac{X^{(k)} \bullet S^{(k)}}{n}$
2. Compute matrices $(X^{(k)})^{-1}$ and $(S^{(k)})^{-1}$, $P^{(k)}$ classically.
3. Obtain classical estimate $\overline{\Delta z}^{(k)}, \overline{\Delta y}^{(k)}$ of $\Delta z^{(k)}, \Delta y^{(k)}$ using IR for the LSP **Quantum-solve**
4. Use classical estimate $\overline{\Delta z}^{(k)}, \overline{\Delta y}^{(k)}$ to obtain classical estimate $\overline{\Delta X}^{(k)}, \overline{\Delta S}^{(k)}$ of $\Delta X^{(k)}, \Delta S^{(k)}$
5. Update current solution

$$X^{(k+1)} \leftarrow X^{(k)} + \overline{\Delta X}^{(k)}, \; S^{(k+1)} \leftarrow S^{(k)} + \overline{\Delta S}^{(k)} \text{ and } y^{(k+1)} \leftarrow y^{(k)} + \overline{\Delta y}^{(k)}$$

$$k \leftarrow k + 1$$

end

Results: Complexity to solve SDO improves to:

$$\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\epsilon}}\left(n^{2.5}\kappa\right)$$

QRAM access and

$$\widetilde{\mathcal{O}}_{n,\kappa,\frac{1}{\epsilon}}\left(n^{4.5}\right)$$

arithmetic operations.

Exponential speedup!

$$\mathcal{O}\left(\frac{n\kappa}{\epsilon}\right)$$

fewer QRAM access than without IR.

Condition number increases

For ease of exposition we consider the case of LO.

- The normal equation system matrix is $A(XS^{-1})A^\top$,
  where $X = \mathrm{diag}(x)$ and $S = \mathrm{diag}(s)$ and $XS^{-1} = \mathrm{diag}\left(\frac{x_1}{s_1}, \ldots, \frac{x_n}{s_n}\right)$
- By complementary slackness either x_i or s_i goes to 0,
  while the other goes to its positive (possibly large) optimal value.
- The condition number is bounded by $\boxed{\kappa = \mathcal{O}\left(\kappa_A \frac{1}{\nu^2}\right)}$
- If we stop IPMs with low prevision, e.g., $\nu > \epsilon = 10^{-2}$, then we have

$$\kappa = \mathcal{O}\left(\kappa_A \frac{1}{\nu^2}\right) = \mathcal{O}\left(\kappa_A \frac{1}{\epsilon^2}\right) \Rightarrow \boxed{\kappa = \mathcal{O}\left(\kappa_A\right)}$$

IR-QIPM with IR for SDO

Input: Problem data $A_1, \dots, A_m, C \in \mathcal{S}^n$, $b \in \mathbb{R}^m$, Error tolerances $0 < \zeta \ll \epsilon < 1$

Output: A ζ-optimal primal-dual solution (X, y, S) to the SDO problem $(A_1, \dots, A_m, b, C)$

Initialize: $X^{(0)} \leftarrow 0$, $y^{(0)} \leftarrow 0$, $\eta^{(0)} \leftarrow 1$, $\varepsilon^{(0)} \leftarrow n$, $k \leftarrow 0$, $\bar{b} \leftarrow b$, $\bar{C} \leftarrow C$

while $\varepsilon > \zeta$ **do**

1. $(\overline{X}, \overline{y}) \leftarrow$ **solve** $(A_1, \dots, A_m, \eta^{(k)}\bar{b}, \eta^{(k)}\bar{C})$ using $O_{\text{SDO}}(\epsilon)$
2. Update solution
$$X^{(k+1)} \leftarrow X^{(k)} + \frac{1}{\eta^{(k)}}\overline{X}, \quad y^{(k+1)} \leftarrow y^{(k)} + \frac{1}{\eta^{(k)}}\overline{y}$$
3. Update refining problem data
$$\bar{b}_i^{(k+1)} \leftarrow b_i - \text{tr}(A_i X^{(k+1)}), \quad \bar{C} \leftarrow C - \sum_{i=1}^{m} y_i^{(k+1)} A_i$$
4. Compute residual $\varepsilon^{(k+1)} = \frac{\text{tr}\left(X^{(k+1)}\bar{C}\right)}{n}$
5. Update scaling factor $\eta^{(k+1)} = \frac{1}{\varepsilon^{(k+1)}}$
6. $k \leftarrow k + 1$

end

Condition number bound:

The condition number bound for the OSS is: $\kappa = \mathcal{O}\left(\kappa_T \frac{1}{\nu}\right) = \mathcal{O}\left(\kappa_T \frac{1}{\epsilon}\right)$

Where $T = \begin{pmatrix} \mathcal{A} & 0 \\ 0 & \mathcal{B}_{\text{Null}} \end{pmatrix}$

IR-QIPM: Complexity for SDO :

At most $\widetilde{\mathcal{O}}_{n,\kappa_T,\frac{1}{\epsilon}}\left(n^{2.5}\kappa_T\right)$

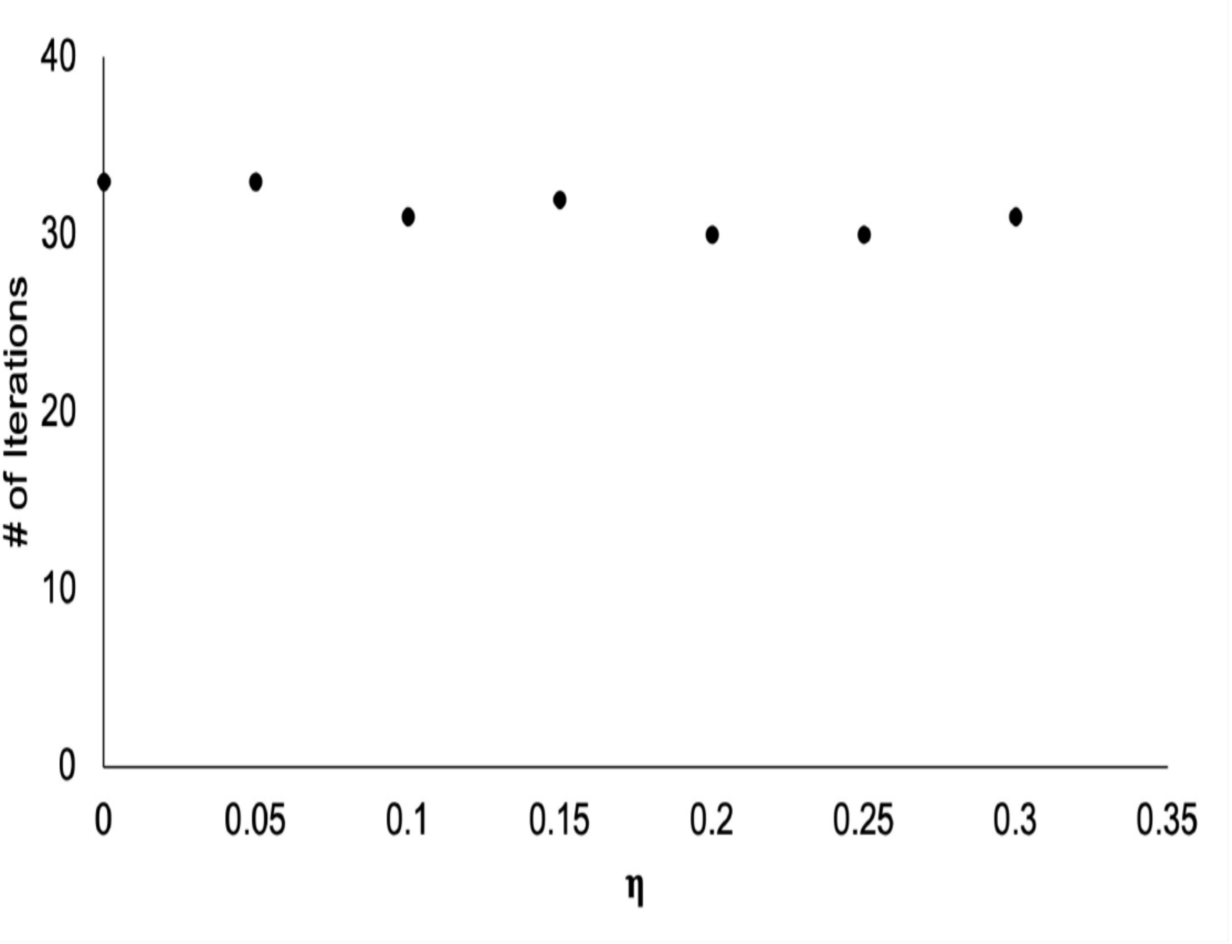
QRAM access and at most $\widetilde{\mathcal{O}}_{n,\kappa_T,\frac{1}{\epsilon}}\left(n^{4.5}\right)$ arithmetic operations.

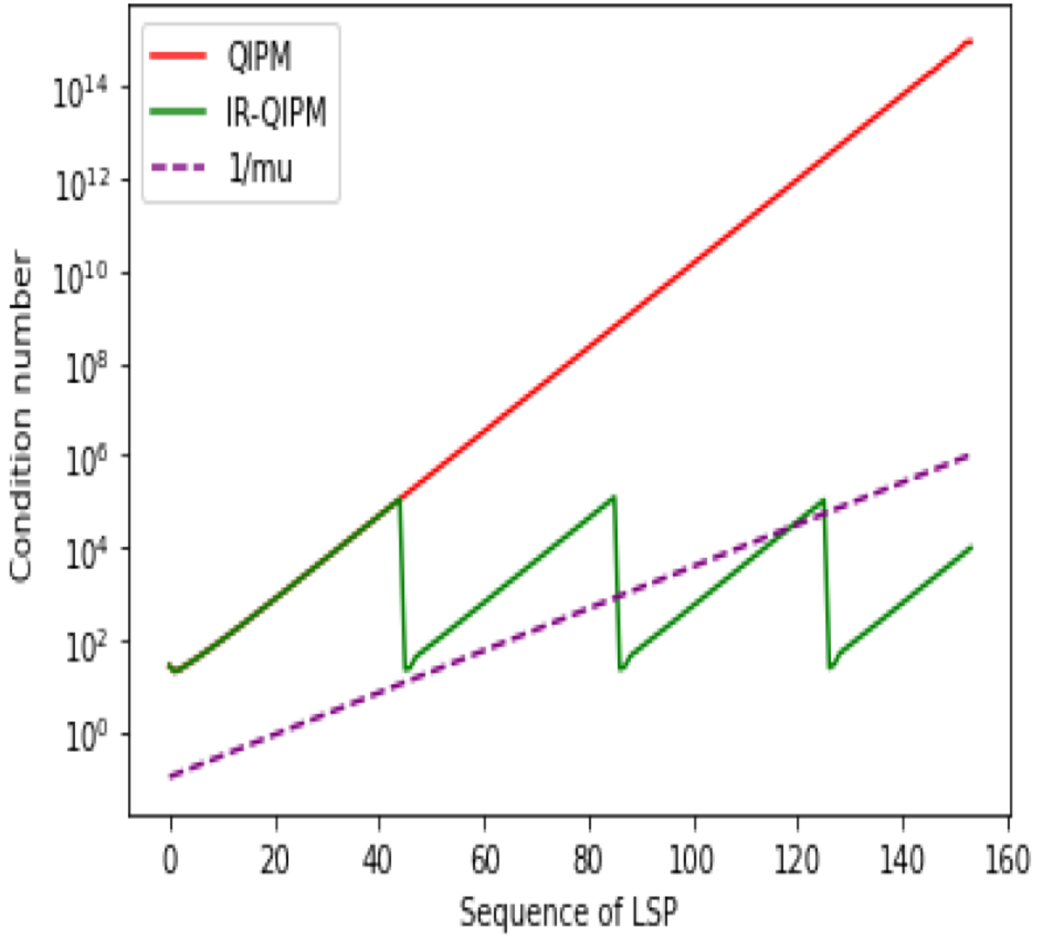
For LO:

At most $\widetilde{\mathcal{O}}_{n,\kappa_T,\frac{1}{\epsilon}}\left(n^{1.5}\kappa_T\right)$ QRAM access, and at most $\widetilde{\mathcal{O}}_{n,\kappa_T,\frac{1}{\epsilon}}\left(n^{2.5}\right)$ arithmetic operations.

Impact of IR on condition number

Developed Python package for IR-IF-QIPMs: https://github.com/QCOL-LU/QIPM





Quadratic Convergence

Quadratic convergence of the optimality gap without any nondegeneracy or strict complementarity assumption!

Theorem:

IR has quadratic convergence toward the optimal solution set of the SDO problem.

$$X^{(k+1)} \bullet S^{(k+1)} \leq \epsilon (X^{(k)} \bullet S^{(k)})^2$$

and

IR obtains an $\tilde{\epsilon}$-optimal solution to the primal-dual SDO in at most

$$\mathcal{O}\left(\log\log\left(\frac{1}{\tilde{\epsilon}}\right)\right)$$

Note: IR needs initial solution at IR each step. A good choice: $(\eta^{(k)} X^{(k)}, 0, \eta^{(k)} S^{(k)})$

IR-IF-QIPMs for LO

Complexity of IF-QIPMs with Iterative Refinement

Algorithm	System	LS Solver	Complexity
Best Theoretical bound	NES	Partial Update	$\mathcal{O}(n^3 L)$
Feasible IPM	NES	Cholesky	$\mathcal{O}(n^{3.5} L)$
II-IPM	NES	PCG	$\mathcal{O}(n^4 L \sqrt{\kappa_M})$
IF-IPM	MNES	PCG	$\mathcal{O}(n^{2.5} L \sqrt{\kappa_M})$
IR-II-IPM	NES	PCG	$\mathcal{O}(n^4 L \kappa_A)$
IR-IF-IPM	MNES	PCG	$\mathcal{O}(n^{2.5} L \kappa_A)$
IR-IF-QIPM	OSS	QTA+QLSA	$\tilde{\mathcal{O}}(n^{2.5} L \kappa_A)$
IR-IF-QIPM	MNES	QTA+QLSA	$\tilde{\mathcal{O}}(n^{2.5} L \kappa_A)$

Polylog factors suppressed

New: *IR-IF-IPMs using PCG to solve Newton Systems*

Analogous properties, complexity as for IR-IF-QIPMs

Quadratic Convergence to the Optimal Set

Further notes on QIPMs

- "Natural" Extensions of II and IF QIPMs:
  - To LO, SDO and SOCO
  - *With and without iterative refinements*
- "Natural" extension of IF-QIPMs:
  - using the Self-Dual Embedding models
  - also: Quantum Dual Log-barrier Method
- Implementation in the Qiskit environment
  - Solving LO problems up to 16 constraints

The future of Optimization

… is bright!

New computing paradigms
New challenges
New opportunities

Bigger impact

Thanks …

Open **for**

Questions and

Discussions

Join and amplify QCOR!

ISE Industrial and Systems Engineering

LEHIGH UNIVERSITY