$$\max_{x \in F} c^T x = \max_{x \in \mathrm{conv}(F)} c^T x$$

Moreover, for almost every $c \in \mathbb{R}^n$ (in the sense of measure) (the) maximizer of the relaxed problem is an extreme point of $\mathrm{conv}(F)$ and therefore $\in F$.

(E.g. Ewald, Larman and Rogers [1970].)

Or, given $x^* \in \mathrm{conv}(F)$ and a suitable description of $\mathrm{conv}(F)$, we can quickly find $\hat{x}$, an extreme point of $\mathrm{conv}(F)$ such that $c^T\hat{x} \geq c^T x^*$.