In this report we see why solving the complementary slackness condition by affine scaling direction (ie solve for $X\,S + X\,dS + dX\,S = 0$ instead of $X\,S + X\,dS + dX\,S = \mu\,I$) is, in the limit, equivalent to solving an ODE with the central path as the stable manifold.

Suppose we start out with some distance from the central path:

$$E_o := X_o\,S_o - \mu_o\,I$$

affine scaling direction with (infinitesimal) step length dt gives

$$\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & A^T & I & 0 \\ S & 0 & X & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} dX \\ dy \\ dS \\ \mathrm{d}(\mu) \end{bmatrix} = -dt \begin{bmatrix} \mathrm{A}(X) - b \\ A^T\,y + S - C \\ X\,S \\ I\,\mu \end{bmatrix}$$

or

$$\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & A^T & I & 0 \\ S & 0 & X & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \frac{dX}{dt} \\ \frac{dy}{dt} \\ \frac{dS}{dt} \\ \frac{\mathrm{d}(\mu)}{dt} \end{bmatrix} = \begin{bmatrix} -\mathrm{A}(X) + b \\ -A^T\,y - S + C \\ -X\,S \\ -I\,\mu \end{bmatrix}$$

The last equation $\frac{\mathrm{d}(\mu)}{dt} = -\mu$ comes from the previous knowledge that $\mu(t) = -K\, e^{(-t)}$

Write out the third equation explicitly:

$$[\frac{dX}{dt}]\, S + X\, [\frac{dS}{dt}] = -X\, S$$

subtract $\frac{\mathrm{d}(\mu)}{dt}$ from both sides

$$[\frac{dX}{dt}]\, S + X\, [\frac{dS}{dt}] - [\frac{\mathrm{d}(\mu)}{dt}]\, I = -X\, S - [\frac{\mathrm{d}(\mu)}{dt}]\, I$$

remembering that $\frac{\mathrm{d}(\mu)}{dt} = -\mu$, we obtain

$$[\frac{dX}{dt}]\, S + X\, [\frac{dS}{dt}] - [\frac{\mathrm{d}(\mu)}{dt}]\, I = -X\, S + \mu\, I$$
$$= -(X\, S - \mu\, I)$$

The LHS can be written as a perfect derivative

$$[\frac{dX}{dt}]\, S + X\, [\frac{dS}{dt}] - [\frac{\mathrm{d}(\mu)}{dt}]\, I = \frac{\mathrm{d}(X\, S - \mu\, I)}{dt}$$

so

$$\frac{\mathrm{d}(X\, S - \mu\, I)}{dt}$$
$$= -(X\, S - \mu\, I)$$

define

$$E := X\,S - \mu\,I$$

we get

$$\frac{dE}{dt} = -E$$

or

$$E = E_o\,e^{(-t)}$$

in other words, the deviation from the central path decays exponentially. Therefore, we conclude that the central path is a stable manifold. This shows that the affine scaling direction is more than a heuristic.