

Robust algorithms for large sparse linear and semidefinite programming

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Current paradigms for search directions for primal-dual interior-point methods for linear programming (LP) and semidefinite programming (SDP) use: (i) form the linearization of the optimality conditions at the current estimate; (ii) form and solve the Schur complement/normal equation for the dual variable \mathbf{d}_y ; (iii) back solve to complete the search direction. These steps result in loss of sparsity and ill-conditioning/instability, in particular when one takes long steps and gets close to the boundary. In addition, for SDP, one usually needs to symmetrize the linearization of the optimality conditions at the current estimate, which introduces further instability. This has resulted in the almost exclusive use of direct, rather than iterative methods, for the linear system.

We look at alternative paradigms based on least squares, an inexact Gauss-Newton approach, and a matrix-free preconditioned conjugate gradient method. This avoids the ill-conditioning in the nondegenerate case. We emphasize exploiting structure in large sparse problems. In particular, we look at large LPs and at SDP relaxations of the: Max-Cut; Quadratic Assignment; Theta function; Nearest Correlation Matrix; and Nearest Euclidean Distance Matrix problems.