MATH 136 **Assignment 7** **Fall/05**

Due: Thursday Nov. 17/05
(Show details of your work. Grade is out of 32.)

1 Linear Independent Sets of Vectors

9 marks

Let V be the vector space of all continuous real valued functions defined on the interval $[0, \pi]$. Consider the following subsets of V. Which of the subsets are linearly independent and why?

1. $$S_1 = \{\sin(t), \cos(t)\}, \quad \text{(two functions)}$$

2. $$S_2 = \left\{\sin^4(t), \cos^4(t), \sin^2(t)\cos^2(t)\right\}, \quad \text{(three functions)}$$

3. $$S_3 = \left\{\sin^4(t), \cos^4(t), \sin^2(t)\cos^2(t), 3.1\right\}, \quad \text{(four functions)}$$

   Note: $f(t) = \sin^2(t)\cos^2(t)$ is a function of t and $g(t) = 3.1$ is also a function of t, i.e. the latter is the constant function that takes the value 3.1 for all t.

2 A Rotation in the Plane

12 marks

Suppose that the vector in the plane $v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is given. Define the transformation T on v to be the clockwise rotation in the plane through an angle $\theta = 45$ degrees, i.e. $T(v)$ is the vector in the plane obtained by rotating v clockwise 45 degrees. Similarly, define the transformation S on v to be the counter-clockwise rotation in the plane through an angle $\theta = 60$ degrees, i.e. $S(v)$ is the vector in the plane obtained by rotating v counter-clockwise 60 degrees.

1. Show that T (and so also S) is a linear transformation and find the matrix representations T_A, T_S of T and S, respectively.

2. What is $W(v) = S(T(v))$? Find a simpler description of the product $W = ST$; and find a matrix representation T_W of W.

3. Confirm that $T_W = T_S T_T$.

3 Page 235, Problem 40

Note: This can be done by hand or with the help of MATLAB. ***4 marks***

4 Page 243, Problem 4

3 marks

5 Page 243, Problem 10

4 marks