CO367 Nonlinear Optimization
Department of Combinatorics and Optimization
University of Waterloo

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NONLINEAR OPTIMIZATION — Assignment 3

Score composition:

(2.16)	(2.21)	(3.18)	(4.17)	(5.1)	(5.5)	(5.13)	(5.26)	Additional Problem	Total
5	5	10	5	20	5	10	15	5	80

(2.21) Some students forgot to mention the trivial case. i.e. when there is no hyperplane separating C and D.

(3.18)
(a)You can use the following identity to solve this problem:
when $|A| < 1$,

$$(I - A)^{-1} = I + A + A^2 + A^3 + ...$$

Therefore, if $Z, V \in \mathbf{S}^{\mathbf{n}}_{++}$, t is very small,

$$\begin{aligned} g(t) &= tr((Z + tV)^{-1}) \\ &= tr((Z^{1/2}(I + tZ^{-1/2}VZ^{-1/2})Z^{1/2})^{-1}) \\ &= tr(Z^{-1/2}(I + tZ^{-1/2}VZ^{-1/2})^{-1}Z^{-1/2}) \\ &= tr(Z^{-1/2}(I - tZ^{-1/2}VZ^{-1/2} + t^2(Z^{-1/2}VZ^{-1/2})^2)Z^{-1/2} + o(t^2)V) \\ &= tr(Z^{-1} - tZ^{-1}VZ^{-1} + t^2(Z^{-3/2}V^2Z^{-3/2})) + o(t^2) \\ &= tr(Z^{-1}) - t * tr(Z^{-1}VZ^{-1}) + t^2 * tr((Z^{-3/2}V^2Z^{-3/2})) + o(t^2) \end{aligned} \tag{1}$$

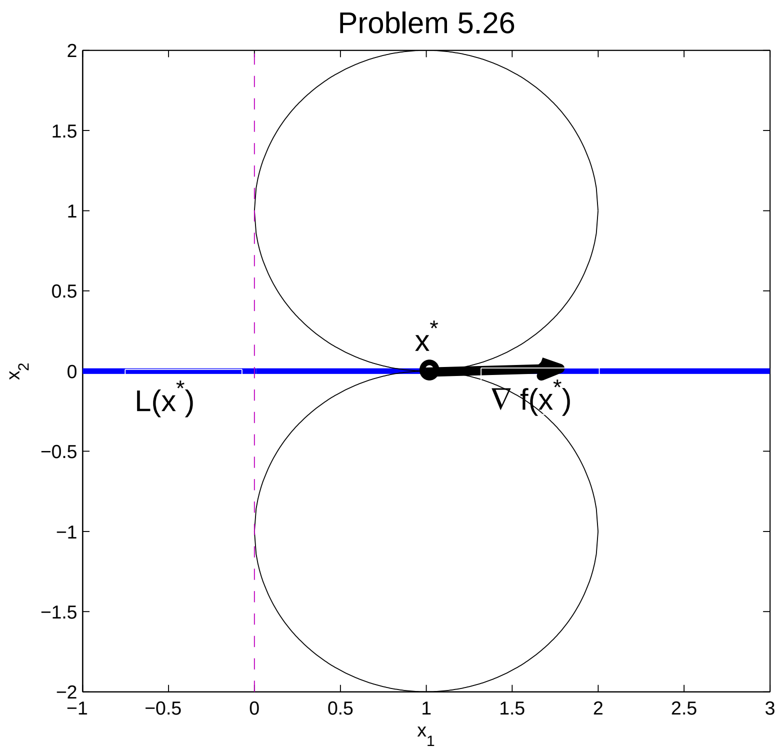
(b) For this problem, it will be helpful if you know (or check) that the geometric mean function:

$$\prod_{i=1}^{n} x_i$$

is concave on $\mathbf{R}^{\mathbf{n}}_{++}$.

(4.17) Notice that because $r_j(x_j)$ is a concave function, then

$$r_j(x_j) = \min(p_j x_j, p_j q_j + p_j^{disc}(x_j - q_j))$$



(5.1)
(c) In order to derive the explicit form of the dual problem, you need to complete the square of the *Lagrangian* and find the hidden constraint. i.e.

$$L(x, \lambda) = (1 + \lambda)(x - \frac{3\lambda}{1+\lambda})^2 - \frac{9\lambda^2}{1+\lambda} + 8\lambda + 1$$

when $\lambda < -1$, $g(\lambda) = -\infty$.

(5.13) The idea for solving this problem is actually very similar to **(5.1)**. You also need to complete the square to obtain the hidden constraint. The main difference is to properly express the problem in matrix form.

(5.26)
(c) In this problem, strong duality doesn't hold. We can see that the feasible set is a single point $x^* = (1, 0)$. Since both constraints are active on this point, the linearizing cone is $L(x^*) = \{x | x_2 = 0\}$, while the tangent cone is $T(x^*) = \{0\}$ (because there is no feasible moving direction from this point). Moreover, the gradient $\nabla f(x^*) = (2, 0)$, which is in $L(x^*)$ but not $T(x^*)$. Therefore, the linearizing cone "improperly" gives an improving direction, and so the KKT condition fails.
However, notice that the Rockafellar-Pshenichni optimality condition holds. (since $\nabla f(x^*)^T(x^* - x) = 0$)

Submitted on March 10, 2011.